

The parity problem in cellular automata

Anna Nenca¹[0000-0003-2746-1061]

Institute of Informatics, Faculty of Mathematics, Physics and Informatics, University of Gdańsk, 80-308 Gdańsk, Poland

The parity problem is one of the classical benchmark problems in cellular automata. The task is to design a local rule that, starting from an arbitrary binary configuration of odd size, converges to the all-0 configuration when the initial number of 1s is even and to the all-1 configuration otherwise.

Although the problem is easy to state, its status remained unclear for many years. This was partly due to the difficulty of computing a global property using only local interactions, and partly because of negative results obtained for related classification problems. In 2013, a major breakthrough was announced with the introduction of the BFO rule, the first proposed solution to the parity problem. However, inconsistencies in the original specification later led to a re-examination of the construction and eventually to a corrected version accompanied by a rigorous proof.

The existence of a solution naturally raises a number of further questions. In particular, it remains unclear whether the parity problem can be solved by a simpler cellular automaton and what the minimum neighborhood size required for such a solution is. Recent advances provide new tools for investigating these questions.