XOR-MAGIC GRAPHS

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We study the construction of xor-magic graphs — connected graphs of order 2^n whose vertices are bijectively labeled with vectors from $\mathbb{F}_2^n$ such that every closed neighborhood sums to zero; such a graph is said to be of power n.

Our main tool is the *self-switching operation*: applying it to an odd xor-magic graph of power n via a binding even subgraph yields one of power $n+1$. We prove that every odd xor-magic graph admits such a subgraph by characterizing non-binding subgraphs as cut subgraphs. We also show that various graph products and Cayley graphs provide systematic methods for constructing xor-magic graphs of higher power and regularity. Combining these tools, we establish the existence of k-regular xor-magic graphs of power n for all $n \geq 3$ and all odd $k \in \{3, 5, \ldots, 2^n - 5\} \cup \{2^n - 1\}$, and prove that no $(2^n - 3)$-regular xor-magic graph of power n exists.

References

[1] A. Batal, On the construction of xor-magic graphs. Discrete Applied Mathematics 379 (2026), 288–315.