

# IMPROPER VERTEX COLORING OF TOROIDAL GRAPHS

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Improper vertex coloring is an assignment of colors to the vertices of a graph in which adjacent vertices do not necessarily receive distinct colors. If we additionally require the subgraphs induced by vertices of the same color to have bounded maximum degree, we speak of defective coloring.

A graph  $G$  is called  $(d_1, \dots, d_k)$ -colorable if its vertex set can be partitioned into subsets  $V_1, \dots, V_k$  such that  $\Delta(\langle V_i \rangle_G) \leq d_i$  for every  $i \in \{1, \dots, k\}$ . The number  $d_i$  is called the defect of the color class  $V_i$ .

While defective colorings of planar graphs have been widely studied, much less is known about graphs embeddable on other surfaces. In this talk, we focus on defective colorings of toroidal graphs. It is known that toroidal graphs are properly 7-colorable (see [1]), and this result is tight, as  $K_7$  is embeddable on the torus. We give a brief overview of known results and present several new ones. In particular, we establish tight upper bounds on color defects for toroidal graphs using  $k$  colors, where  $k \in \{4, 5, 6\}$ , and discuss some related results.

## References

- [1] P. J. Heawood, Map-colour theorem. *Quart. J. Pure Appl. Math.*, 24:332–338, 1890.