RECOLORING GRAPHS OF SOME 2- AND 3-DICHROMATIC DIGRAPHS

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The *k-recoloring graph* of a loopless digraph D, denoted by $\mathcal{D}_k(D)$ is the graph whose vertices correspond to the acyclic k-colorings of D, where two colorings are adjacent if they differ in the color assigned to exactly one vertex. Introduced by Bousquet *et al.* (2024), this notion extends the widely studied recoloring graph of an undirected graph; see Mynhardt and Nasserasr (2020) for a survey. Recent work by Bousquet *et al.* and Picasarri-Arrieta (2024) has established sufficient conditions for the connectivity of $\mathcal{D}_k(D)$ based on maximum and minimum degrees and semi-degrees, alongside upper bounds on its diameter, and proposed several open problems.

For undirected graphs, a classic result by Cereceda (2007) shows that the recoloring graph of a graph G is disconnected when $k = \chi(G) \in \{2, 3\}$, while for $k = \chi(G) \geq 4$, both connected and disconnected examples may occur. Motivated by this, we investigate the behavior of $\mathcal{D}_k(D)$ when k equals the dichromatic number $\vec{\chi}(D)$. In this talk, we present new structural results concerning the connectivity and properties of the recoloring graphs for specific families of 2- and 3-dichromatic digraphs.

References

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