CHARACTERIZATION OF GRAPHS WITH ORIENTABLE TOTAL DOMINATION NUMBER EQUAL TO $|V| - 1$

Zoltán L. Blázsik

Bolyai Institute (University of Szeged), and University of Johannesburg

e-mail: blazsik@server.math.u-szeged.hu

Leila Vivien Nagy

Eötvös University

In a directed graph D, a vertex subset $S \subseteq V$ is a total dominating set if every vertex of D has an in-neighbor from S. A total dominating set exists if and only if every vertex has at least one in-neighbor. We call the orientation of such directed graphs valid. The total domination number of D, denoted by $\gamma_t(D)$, is the size of the smallest total dominating set of D. For an undirected graph G, we investigate the upper (or lower) orientable total domination number of G, denoted by $\mathrm{DOM}_t(G)$ (or $\mathrm{dom}_t(G)$), that is the maximum (or minimum) of the total domination numbers over all valid orientations of G. We characterize those graphs for which $\mathrm{DOM}_t(G) = |V(G)| - 1$, and consequently we show that there exist infinite families of graphs for which $\mathrm{DOM}_t(G) = |V(G)| - 1$, meanwhile $\mathrm{dom}_t(G) = 3$ that maximizes both $\mathrm{DOM}_t(G) - \mathrm{dom}_t(G)$ and $\frac{\mathrm{DOM}_t(G)}{\mathrm{dom}_t(G)}$.

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