# ON EXTERNALLY SUPPORTED INDEPENDENT NUMBER OF GRAPHS

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Let $G$ be a graph. A set $D \subseteq V(G)$ is an *isolating set* of $G$ if only isolated vertices remain in $G$ outside of closed neighborhoods centered in vertices of $D$. The *isolating number* $\iota(G)$ of $G$ is the minimum cardinality of an isolating set of $G$. A trivial upper bound that depends on independence number $\alpha(G)$ and maximum degree $\Delta(G)$ is

$$\iota(G) \geq \left\lceil \frac{|V(G)| + 1 - \alpha(G)}{\Delta(G) + 1} \right\rceil .$$

This bound is sharp for a small class of graphs because the vertices that remain usually do not form an independent set of cardinality $\alpha(G)$. Therefore we introduce the *externally supported independent number* $\alpha_{\rm es}(G)$ of a graph $G$ as the maximum cardinality of an independent set $B$ with an additional condition, that vertices from $N(B)$ are dominated by vertices in $V(G) - N[B]$ (which form an isolating set of $G$). This parameter yields an improved upper bound on the isolation number $\iota(G)$. We show that computing $\alpha_{\rm es}(G)$ in NP-hard, while for trees we present a linear-time algorithm. We also establish several sharp bounds on $\alpha_{\rm es}(G)$ for general graphs, with additional refined results for trees. In several cases, we completely describe the extremal graph classes attaining these bounds.

This is a join work with Dragana Božović, Adriana Roux and Aleksandra Tepeh

**Keywords:** externally supported independent number, independent number, isolating number, trees.

**AMS Subject Classification:** 05C69.