ISOLATION OF 3-VERTEX PATHS

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The 3-path isolation number $\iota(G, P_3)$ of a connected graph G is the size of a smallest subset D of the vertex set of G such that removing the closed neighbourhood $N[D]$ of D from G yields a graph consisting only of independent vertices and independent edges.

Zhang and Wu [4] proved the sharp upper bound $\iota(G, P_3) \leq \frac{2n}{7}$ for a connected n-vertex graph G which is not a 3-path or a 3-cycle or a 6-cycle. This bound is attained by infinitely many graphs containing induced 6-cycles. Huang, Zhang, and Jin [3] proved that if G has no 6-cycles, or G has no induced 5-cycles and no induced 6-cycles, then $\iota(G, P_3) \leq \frac{n}{4}$ unless G is a 3-path or a 3-cycle or a 7-cycle or an 11-cycle. They asked if the bound still holds asymptotically for connected graphs having no induced 6-cycles.

We recently solved the problem in its entirety. We first showed in [1] that if G is subcubic and has no induced 6-cycles, then $\iota(G, P_3) \leq \lfloor \frac{n}{4} \rfloor$ unless G is a copy of one of 12 graphs whose orders are 3, 7, 11, and 15. Furthermore, we showed in [2] that if G is a connected n-vertex graph that has no induced 6-cycles, then $\iota(G, P_3) \leq \lfloor \frac{n+1}{4} \rfloor$. This bound is attainable for every n. The bound $\frac{n+1}{4}$ is attained by infinitely many graphs G with $\Delta(G) \leq 4$. We also showed that if $\Delta(G) \geq 5$, then $\iota(G, P_3) < \frac{n+1}{4}$.

References

[1] K. Bartolo, P. Borg, and D. Scicluna, Solution to a 3-path isolation problem for subcubic graphs. Discrete Math. 349(5) (2026), paper 114970.

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[3] Y. Huang, G. Zhang, X. Jin, New results on the 1-isolation number of graphs without short cycles. Discrete Appl. Math. 379 (2026) 222–235.

[4] G. Zhang and B. Wu, $K_{1,2}$-isolation in graphs. Discrete Applied Mathematics 304 (2021), 365–374.