

THE $L(2, 1)$ -GRUNDY NUMBER

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Graph colorings with distance conditions have received considerable attention due to their theoretical interest and their applications in problems such as frequency assignment in radio networks. One of the most studied examples is the $L(h, k)$ -coloring, introduced by Griggs and Yeh [1, 2], in which the vertices of a graph are colored with non-negative integers so that adjacent vertices differ by at least h , while vertices at distance two differ by at least k . The special case $L(2, 1)$ has been particularly studied, as it models interference conditions in wireless communication systems.

The classical $L(2, 1)$ -chromatic number, denoted by $\lambda_{2,1}(G)$, is the minimum possible span among all $L(2, 1)$ -colorings of a graph G . Since computing this number is NP-complete [1], greedy algorithms provide a natural heuristic approach: vertices are colored one by one in a fixed order, assigning to each vertex the smallest available color satisfying the $L(2, 1)$ -constraints.

However, the performance of this greedy procedure may vary significantly depending on the chosen vertex ordering. This leads to a natural question: how bad can a greedy $L(2, 1)$ -coloring be compared with an optimal one? Motivated by this question, we introduce the $L(2, 1)$ -Grundy number $\Gamma_{2,1}(G)$, a new parameter that captures the worst-case performance of greedy algorithms for $L(2, 1)$ -colorings. This parameter generalizes the classical Grundy number for proper colorings to the $L(2, 1)$ colorings and provides a safety bound for applications such as frequency assignment, task scheduling, and resource allocation.

References

- [1] J. R. Griggs, R. K. Yeh, Labeling graphs with a condition at distance 2. *SIAM J. Discrete Math.* 5, (1992) 586–595.
- [2] R. K. Yeh, Labeling Graphs with a Condition at Distance Two. PhD Thesis, University of South Carolina, Columbia, South Carolina, 1990.