SHORTEST PATHS IN MULTIMODE GRAPHS

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In this work we study shortest path problems in multimode graphs, a generalization of the min-distance measure introduced by Abboud, Vassilevska W. and Wang [1]. A multimode shortest path is the shortest path using one of multiple 'modes' of transportation that cannot be combined. This represents real-world scenarios where different modes are not combinable, such as flights operated by different airline alliances. The problem arises naturally as well in machine learning in the context of learning with multiple embedding. More precisely, a k-multimode graph is a collection of k graphs on the same vertex set and the k-mode distance between two vertices is defined as the minimum among the distances computed in each individual graph.

We focus on approximating fundamental graph parameters on these graphs, specifically diameter and radius. In undirected multimode graphs we first show an elegant linear time 3-approximation algorithm for 2-mode diameter. We then extend this idea into a general subroutine that can be used as a part of any α-approximation, and use it to construct a 2 and 2.5 approximation algorithm for 2-mode diameter. For undirected radius, we introduce a general scheme that can compute a 3-approximation of the k-mode radius for any k and runs in near linear time in the case of $k = O(1)$. In the directed case we establish an equivalence between approximating 2-mode diameter on DAGs and approximating the min-diameter, while for general graphs we develop novel techniques and provide a linear time algorithm to determine whether the diameter is finite.

We also develop many conditional fine-grained lower bounds for various multimode diameter and radius approximation problems. We are able to show that many of our algorithms are tight under popular fine-grained complexity hypotheses, including our linear time 3-approximation for 3-mode undirected diameter and radius. As part of this effort we propose the first extension to the Hitting Set Hypothesis [1], which we call the ℓ-Hitting Set Hypothesis. We use this hypothesis to prove the first parameterized lower bound tradeoff for radius approximation algorithms.

References

[1] Amir Abboud, Virginia Vassilevska Williams, and Joshua Wang. Approximation and fixed parameter subquadratic algorithms for radius and diameter in sparse graphs. In Proceedings of the Twenty-Seventh Annual ACM-SIAM Symposium on Discrete Al- gorithms, SODA '16, page 377–391, USA, 2016. Society for Industrial and Applied Mathematics.