PACKING LIST-COLORINGS AND PROPER-PATH COLORINGS

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We introduce and study the *packing proper connection number*, a list-packing analogue of the proper connection number of a graph. Given an edge k-list assignment L, a proper-connected L-packing is a collection of size k of pairwise disjoint proper-connected colorings chosen from the prescribed lists. Here, a proper-connected coloring is an edge-coloring such that between every pair of distinct vertices there exists a properly colored path. The minimum list size guaranteeing the existence of such a packing is called the packing proper connection number and is denoted by $pc^*_\ell(G)$.

We prove that every 2-connected graph has packing proper connection number at most 3. We also show that this bound is sharp by constructing a bipartite 2-connected graph with packing proper connection number equal to 3. Extending this result, we prove that every connected bridgeless graph also has packing proper connection number at most 3.

We further introduce the notion of a *one-path proper-connected packing*, in which for every pair of vertices there exists a single path that is properly colored simultaneously in all packing layers. For this stronger variant, we prove that every 2-connected graph admits such a packing from arbitrary edge list assignments of size 4. More generally, for a connected bridgeless graph with maximum block degree l, we establish an upper bound depending only on l for the corresponding one-path packing proper connection number.