# DISJOINT CORRESPONDENCE COLORINGS FOR $K_5$-MINOR-FREE GRAPHS

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Thomassen famously proved that every planar graph is 5-choosable. We explore variants of this result, focusing on finding disjoint correspondence colorings, in the more general class of $K_5$-minor-free graphs. Correspondence colorings generalize list colorings as follows. Given a graph $G$ and a positive integer $t$, a correspondence $t$-cover $\mathbf{M}$ assigns to each $v \in V(G)$ a set of allowable colors $\{1_v, \ldots, t_v\}$ and to each edge $vw \in E(G)$ a matching between $\{1_v, \ldots, t_v\}$ and $\{1_w, \ldots, t_w\}$. An $\mathbf{M}$-coloring $\varphi$ picks for each vertex $v$ a color $\varphi(v)$ (from the set $\{1_v, \ldots, t_v\}$) such that for each edge $vw \in E(G)$ the colors $\varphi(v), \varphi(w)$ are not matched to each other. Two $\mathbf{M}$-colorings $\varphi_1, \varphi_2$ of $G$ are *disjoint* if $\varphi_1(v) \neq \varphi_2(v)$ for all $v \in V(G)$.

For every $K_5$-minor-free graph $G$ and every correspondence 6-cover $\mathbf{M}$ of $G$, we construct 3 pairwise disjoint $\mathbf{M}$-colorings $\varphi_1, \varphi_2, \varphi_3$. In contrast, we provide examples of $K_5$-minor-free graphs and correspondence 5-covers $\mathbf{M}$ that do not admit 3 disjoint $\mathbf{M}$-colorings.