PROPER AND PERFECT SECONDARY DOMINATING SETS IN GRAPHS

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Let $k \geq 1$ be an integer. A subset $D \subseteq V(G)$ is $(1,k)$-*dominating* if for every vertex $v \in V(G) \setminus D$ there exist distinct vertices $u, w \in D$ such that $uv \in E(G)$ and $d_G(v,w) \leq k$. For $k = 1$ we obtain the notion of $(1,1)$-dominating sets, also known as *2-dominating sets*, while $k = 2$ leads to the concept of $(1,2)$-*dominating sets*.

In [3], Michalski et al. introduced the concept of *proper* $(1,2)$-*dominating sets* in order to distinguish $(1,2)$-dominating sets from $(1,1)$-dominating ones. A proper $(1,2)$-dominating set D is a $(1,2)$-dominating set that is not $(1,1)$-dominating - so there must be a vertex outside D that has exactly one neighbour in D.

Building on this idea, Bednarz and Pirga [2] defined *perfect* $(1,2)$-*dominating sets* by imposing the additional requirement that every vertex outside the set must have exactly one neighbour in it. The problem of existence of such sets and its computational complexity were studied in [1], where it was shown that almost all subcubic graphs admit a perfect (1,2)-dominating set, whereas deciding whether a 4-regular graph has one is NP-complete.

In this talk several properties of proper and perfect $(1,2)$-dominating sets will be presented.

References

[1] U. Bednarz, J. Kratochvíl, A. Michalski, Complexity of Perfect (1, 2)-Dominating Sets in Low-Degree Graphs. In: Di Giacomo, E., Mondal, D. (eds) WALCOM: Algorithms and Computation. WALCOM 2026. Lecture Notes in Computer Science 16444, 203–214. Springer, Singapore,.

[2] U. Bednarz, M. Pirga, (1, 2)-PDS in graphs with the small number of vertices of large degrees. Opuscula Mathematica 45(1) (2025), 53–62.

[3] A. Michalski, I. Włoch, M. Dettlaff, M. Lemańska, On proper (1,2)-dominating sets. Mathematical Methods in the Applied Sciences 45(11) (2022), 7050–7057.