GENERALIZED MAJORITY AND QUASI-MAJORITY NEIGHBOR SUM DISTINGUISHING EDGE-COLORINGS OF GRAPHS

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An edge-coloring c of a graph G induces a vertex-coloring $\sigma_c : V(G) \to \mathbb{N}$ by $\sigma_c(v) = \sum_{u \in N_G(v)} c(vu)$ for each vertex $v \in V(G)$. If $\sigma_c(u) \neq \sigma_c(v)$ for every edge $uv \in E(G)$, then c is called neighbor sum distinguishing (NSD for short). This notion is related to the 1-2-3 Conjecture proved by Keusch [4].

Several variants of NSD edge-colorings arise by imposing additional restrictions on edge-colorings. Flandrin et al. [2] introduced NSD proper edge-colorings. In [3], we investigated majority and quasi-majority NSD edge-colorings, in which, for every vertex $v \in V(G)$, at most $\frac{d(v)}{2}$ and $\left\lceil \frac{d(v)}{2} \right\rceil$ of the edges incident to v, respectively, have the same color.

We study generalizations of the above edge-colorings. For $k \in \mathbb{N}$, Bock et al. [1] introduced $\frac{1}{k}$-majority edge-colorings, where each color appears on at most $\frac{d(v)}{k}$ edges incident to each vertex $v \in V(G)$. We investigate their NSD variants, as well as the quasi-majority version, where each color appears on at most $\left\lceil \frac{d(v)}{k} \right\rceil$ edges incident to each vertex $v \in V(G)$.

References

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