

# SEMI-MAGIC SQUARES OVER DIHEDRAL GROUPS

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A *magic square of side  $n$*  is an  $n \times n$  array whose entries are all elements of  $[n^2]$ , each of them present once, such that the sum of every row, column, main and backward main diagonal is equal to the same number  $\mu = n(n^2 + 1)/2$ . Magic squares are some of the oldest known math structures and have been studied for thousands of years. They are known to exist for any  $n \neq 2$ .

When we fill the entries with elements of a group  $\Gamma$  of order  $n^2$  instead, we speak about a  $\Gamma$ -*magic square of side  $n$* .

Until about two years ago, the only result in this direction was a construction of  $Z_n \oplus Z_n$ -magic squares [2]. We recently proved [1] that a  $\Gamma$ -magic square of side  $n$  exists for any Abelian group of order  $n^2$  if and only if  $n \neq 2$ .

With Abelian groups completely characterized, we turned to non-Abelian groups. The simplest such group is the *dihedral group*  $D_m$  of order  $2m$  with elements  $r^i s^j$ ,  $i \in [0, m-1]$ ,  $j \in [0, 1]$ , defined by identities  $r^0 = r^m = e$ ,  $s^2 = e$ , and  $r^i s = s r^{-i}$ .

We found  $\Gamma$ -semimagic squares of side  $2n$  for dihedral groups  $D_{2n^2}$  of order  $4n^2$  for every  $n > 1$ . (A *semimagic square* only requires the row and column sums to be all equal, not necessarily the diagonals.)

I will present three different constructions for the dihedral group  $D_{2n^2}$ .

## References

- [1] S. Cichacz, D. Froncek, Magic squares on Abelian groups, *Discrete Math.* **349(7)** (2026), 115033.
- [2] H. Sun, W. Yihui, Note on Magic Squares and Magic Cubes on Abelian Groups, *J. Math. Res. Exposition* **17(2)** (1997), 176–178. (2012), 1125–1132.