

# NEW CONSTRUCTIONS OF GRACEFUL GRAPH LABELINGS

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Let  $G = (V, E)$  be a graph with  $n$  vertices and  $m$  edges. We say that an injective mapping  $f : V \rightarrow \{0, 1, \dots, m\}$  is a *graceful labeling* of  $G$  if the induced edge labeling  $f^* : E \rightarrow \mathbb{N}$ , given by  $f^*(vw) = |f(v) - f(w)|$ , is injective and  $\text{Im}(f^*) = \{1, \dots, m\}$ .

The interesting problem of graceful graph labeling was first introduced by Rosa in [1]. A major open problem is the *Ringel–Kotzig conjecture*, which states that all trees are graceful. Later on, variants of graceful labelings with additional, more restrictive properties have been introduced and studied. These additional constraints and new notions have made the field more lively and interesting. This topic has been the focus of many papers; an overview of numerous results can be found in the well-known dynamic survey by Gallian [2].

In this talk, we present our new results on two such notions:  $\alpha$ -labeling and edge-odd graceful labeling of a graph.

An  $\alpha$ -*labeling* of  $G$  is a graceful labeling  $f$  with the property that there exists a constant  $\alpha \in \mathbb{N}$  such that for every edge  $vw \in E$ , either  $f(v) \leq \alpha < f(w)$  or  $f(w) \leq \alpha < f(v)$ . We present a complete characterization of  $\alpha$ -labelable  $B(n, k)$  trees, which are constructed by connecting  $n$  star graphs  $S_k$  to a root vertex. We completely resolve this problem by showing that a  $B(n, k)$  tree is  $\alpha$ -labelable if and only if  $k \geq \frac{n}{2} + 1$  or  $k = 1$ .

An *edge-odd graceful labeling* of  $G$  is an injective mapping  $f : E \rightarrow \{1, 3, 5, \dots, 2m - 1\}$  such that the induced mapping  $f^+ : V \rightarrow \mathbb{N}$ , where each vertex is assigned the sum of all incident edge labels (mod  $2m$ ), is injective. We show that the complete graph  $K_n$  is edge-odd graceful for every  $n$ . We also show that the complete bipartite graph  $K_{r,s}$ , where  $r$  and  $s$  are coprime and  $r, s \neq 1$ , is edge-odd graceful.

## References

- [1] A. Rosa, On certain valuations of the vertices of a graph, In: Theory of graphs (Internat. Sympos., Rome, 1966), 349–355, Gordon & Breach, New York, 1967.

- [2] J. A. Gallian, A Dynamic Survey of Graph Labeling, *Electron. J. Comb.*, #DS6, 1997.