BOUNDS AND EXACT VALUES OF THE RAINBOW-K DOMINATION NUMBER FOR SELECTED GRAPH FAMILIES

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Rainbow-k domination is a graph domination model in which each vertex is either left uncolored or assigned exactly one of k colors, and every uncolored vertex must see all k colors in its open neighborhood. The corresponding parameter, denoted by $\widetilde{\gamma}_k(G)$, is the minimum number of colored vertices in such an assignment. This notion is a natural restriction of classical k-rainbow domination and can be interpreted as a model for placing k distinct types of resources under the constraint that each location may host at most one resource type.

In this talk, we present bounds and exact results for the rainbow-k domination number in several graph classes. We first discuss general lower and upper bounds, including degree-based estimates, bounds expressed through sparse induced subgraphs, and probabilistic upper bounds. These results show how $\widetilde{\gamma}_k(G)$ is influenced both by local degree constraints and by the structure of the subgraph induced by uncolored vertices.

We then turn to exact values for selected families. For cycles, the rainbow-k domination number can be determined completely. In the cubic setting, we present an exact formula for the rainbow-3 domination number of Möbius ladders, where the value depends on the order of the graph modulo 12. We also study generalized Petersen graphs $P(6t, t)$. Using their natural block structure, we obtain an exact dynamic-programming algorithm which computes $\widetilde{\gamma}_k(P(6t, t))$ in linear time with respect to t, for every fixed number of colors k. The resulting computations, supported by integer linear programming, reveal strong periodic behavior and lead to conjectural closed formulas for the cases $k = 2$ and $k = 3$.

The talk concludes with a discussion of the patterns suggested by these results and the open problems they motivate. The obtained results show that, despite its simple local definition, the parameter exhibits rigid global behavior on highly structured graph families and leads to natural questions about periodicity and extremal configurations.

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