

# Coloring Mixed Graphs

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## Abstract

A mixed graph is a graph containing both undirected and directed edges. A proper coloring of a mixed graph is an assignment of positive integers to the vertices such that the endpoints of every undirected edge receive distinct colors, and for every directed edge  $(u, v)$ , the color assigned to  $u$  is smaller than the color assigned to  $v$ .

Such colorings arise naturally in problems where one has to assign levels, labels, or time slots subject to both separation and precedence constraints. Examples include scheduling, timetabling, ranking, graph drawing, and related combinatorial optimization problems.

The corresponding decision problem is NP-complete, even for interval mixed graphs. In this talk, we introduce the problem of coloring mixed graphs, discuss its basic properties and applications, and present two exact algorithms for computing the minimum number of colors in a proper coloring. The first algorithm uses exponential space, whereas the second works in polynomial space. We also analyze their complexity and describe some connections with problems in extremal graph theory.