

# Zero tolerance for non-zero sums: additive graph decompositions

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## Abstract

The intriguing concept of additive 2-designs introduced in [4] has opened the door to the more general notion of “additive combinatorial designs” [2]. These are point-block incidence structures where the points are elements of an abelian group  $G$ , and the sum of the points in each block is zero in  $G$ . This explains the title of the talk: in this context, only zero-sum blocks are tolerated. This requirement is such a severe constraint that constructing these additive structures appears, in general, to be a highly challenging problem.

In this talk, I will shift the focus toward the graph-theoretical perspective, presenting a selection of results concerning “additive graph decompositions” with a particular emphasis on additive Steiner 2-designs [1, 2]. Indeed, a Steiner 2-design can be viewed as a decomposition of a “large” complete graph into copies of a “smaller” complete graph. It should be underlined, however, that the literature contains several constructions of other types of combinatorial designs that prove to be additive, even when such a property was not the primary focus of the authors [3, 5].

## References

- [1] M. Buratti, M. Galici, A. Montinaro, A. Nakic, A. Wassermann, *EA(q)-additive Steiner 2-designs*, preprint.
- [2] M. Buratti, F. Merola, A. Nakic, *Additive combinatorial designs*, Des. Codes Cryptogr. **93** (2025), 2717–2740.
- [3] M. Buratti, A. Pasotti, *Heffter spaces*, Finite Fields Appl. **98** (2024), Article number 102464.
- [4] A. Caggegi, G. Falcone, M. Pavone. On the additivity of block designs. J. Algebr. Comb. **45** (2017), 271–294.
- [5] S. Chicacz, D. Froncek, *Magic squares on Abelian groups*, Discrete Math. **349** (2026), Article number 115033.