# Independent domination in 3-regular graphs and domination in 4-regular graphs

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A set $S$ of vertices in a graph $G$ is a dominating set of $G$ if every vertex not in $S$ is adjacent to a vertex in $S$. An independent dominating set in $G$ is a dominating set of $G$ with the additional property that it is an independent set. The domination number, $\gamma(G)$, and the independent domination number, $i(G)$, are the minimum cardinalities among all dominating sets and independent dominating sets in $G$, respectively. The $\frac{3}{8}$-conjecture for independent domination states that if $G$ is connected 3-regular graph of order $n$ different from $K_{3,3}$ and the 5-prism $C_5 \,\square\, K_2$, then $i(G) \le \frac{3}{8}n$. The $\frac{1}{3}$-conjecture for domination states that if $G$ is a 4-regular graph of order $n$, then $\gamma(G) \le \frac{1}{3}n$. We prove the $\frac{3}{8}$-conjecture for independent domination (see [1]), and we prove the $\frac{1}{3}$-conjecture for domination when the 4-regular graph has no induced 4-cycle (see [1]). A thorough treatise on dominating sets can be found in [2].

## References

[1] B. Brešar, T. Dravec, and M. A. Henning, A proof of the $\frac{3}{8}$-conjecture for independent domination in cubic graphs, mansucript.

[2] T. W. Haynes, S. T. Hedetniemi, and M. A. Henning, *Domination in Graphs: Core Concepts* Series: Springer Monographs in Mathematics, Springer, Cham, 2023. xx + 644 pp.

[3] M. A. Henning and A. Yeo, Domination in 4-regular graphs with no induced 4-cycles, mansucript.