ROMAN DOMINATION ON FUZZY GRAPHS

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We make a contribution to the well-known problem of Roman domination in graph theory as it relates to fuzzy graphs. Domination in fuzzy graphs has been studied using a variety of approaches. By taking into account just effective edges, Somasundaram and Somasundaram [1] examined domination and total domination in fuzzy graphs. Domination in fuzzy graphs was introduced by Nagoor Gani and Chandrasekaran [2] as the number of vertices in a dominating set that makes use of strong edges. Based on the weight of strong edges, Manjusha and Sunitha [4] determined the domination number of fuzzy graphs. Moreover, they used this idea to examine a fuzzy graph's strong node covering number [5].

We will use the weights of strong edges to define the Roman domination number for a fuzzy graph, based on the domination concept put forth by Manjusha and Sunitha. Cockayne et al. [3], who took their cue from a historical defensive tactic ascribed to the reign of Emperor Constantine I The Great (see [6]), are credited for establishing Roman dominance in graphs. This tactic required that every weak point in the Roman Empire have a neighboring fortress (having two legions) that could send a legion to defend it in case of an unexpected attack. This guaranteed that the more powerful city would not have to jeopardize its own security in order to send reinforcements to defend the beleaguered area.

This paper presents the notions of strong-neighbors Roman domination function/number of a fuzzy graph and shows how it relates to other well-known domination parameters. For particular fuzzy graphs, we derive bounds, find the strong-neighbors Roman domination number, and describe the fuzzy graphs for which extreme values are obtained.

References

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