THE ALMOST MAJORITY NEIGHBOR SUM DISTINGUISHING EDGE COLORING

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A k-edge coloring of a graph with colors in $[k]$ is neighbor sum distinguishing if, for any two adjacent vertices, the sums of the colors of the edges incident with each of them are distinct. The smallest value of k such that a neighbor sum distinguishing k-coloring of G exists is denoted by $\chi^e_{\sum}(G)$. When we add the additional restriction that the edge k-coloring must be proper, then the smallest value of k such that such a coloring exists is denoted by $\chi'_{\sum}(G)$. The first type of coloring is related with the 1-2-3 Conjecture, which was already proven by Keusch ([2]). The second type of coloring is related with another conjecture proposed by Flandrin et al. ([1]), which states that $\chi'_{\sum}(G) \leq \Delta(G) + 2$ for any graph G with no components isomorphic to K_2 and $G \neq C_5$. This conjecture remains open.

We consider an edge coloring that is on one hand stronger than the edge coloring in the 1-2-3 Conjecture, and on the other hand weaker than the coloring in conjecture proposed by Flandrin et al.. An edge k-coloring of a graph G is called almost majority if for every vertex $v \in V(G)$ and every color $\alpha \in [k]$ at most $\lceil d(v)/2 \rceil$ edges incident to v have the color α. An edge k-coloring of a graph G is called almost majority neighbor sum distinguishing if it is almost majority and neighbor sum distinguishing. The minimum value of k for which there exists such an edge k-coloring of a graph G is called the almost majority neighbor sum distinguishing index of a graph G and is denoted by $\chi^{AM}_{\sum}(G)$. We study $\chi^{AM}_{\sum}(G)$ for some classes of graphs.

References

[1] E. Flandrin, A. Marczyk, J. Przybyło, J-F. Sacle, M. Woźniak, Neighbour sum distinguishing index. Graphs Combin. 29(5) (2013), 1329–1336.

[2] R. Keusch, A Solution to the 1-2-3 Conjecture. J. Combin. Theory Ser. B 166 (2024) 182–202.