PATH ISOLATION IN GRAPHS

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Given a graph G and a set $\mathcal{F}$ of graphs, the $\mathcal{F}$-isolation number is the size of a smallest subset D of the vertex set of G such that $G - N[D]$ (the graph obtained from G by removing the closed neighbourhood of D) does not contain a copy of a graph in $\mathcal{F}$. The path isolation number $\iota(G, P_i)$ for $i > 0$ has attracted particular interest among graph theorists. For $i = 1$, since $P_1 = K_1$, we have Ore's (1962) result [4] that $\gamma(G) = \iota(G, P_1) \leq \frac{n}{2}$ where $\gamma(G)$ is the domination number of an n-vertex connected graph G. For $i = 2$, since $P_2 = K_2$, we have Caro and Hansberg's (2017) result [2] that $\iota(G, P_2) \leq \frac{n}{3}$ provided G is connected but not a 5-cycle or a 2-clique. For $i = 3$, it was shown by Zhang and Wu [5], and independently and in a stronger form by Borg [1], that $\iota(G, P_3) \leq \frac{2n}{7}$ unless $G \in \{P_3, C_3, C_6\}$. This can be improved to $\frac{n}{4}$ if G is not a $\{P_3, C_7, C_{11}\}$-graph and the girth is at least 7. Recently Huang, Zhang and Jin [3] showed that for a connected graph G that has no 6-cycles or has no induced 5- and 6-cycles, then $\iota(G, P_3) \leq \frac{n}{4}$ provided G is not a $\{P_3, C_3, C_7, C_{11}\}$-graph. Joint work with Bartolo and Borg improved this result for subcubic graphs. If G is subcubic and has no induced 6-cycles, then $\iota(G, P_3) \leq \frac{n}{4}$ provided G is not one of twelve specific graphs. The bound is sharp.

References

[1] P. Borg, Isolation of connected graphs, Discrete Appl. Math. 339 (2023), 154–165.

[2] Y. Caro and A. Hansberg, Partial domination - the isolation number of a graph, Filomat 31:12 (2017), 3925–3944.

[3] Y. Huang, G. Zhang and X. Jin, New results on the 1-isolation number of graphs without short cycles, arXiv:2308.00581.

[4] O. Ore, Theory of graphs, American Mathematical Society Colloquium Publications, vol. 38, American Mathematical Society, Providence, R.I., 1962.

[5] G. Zhang and B. Wu, $K_{1,2}$-isolation in graphs, Discrete Applied Mathematics 304 (2021), 365–374.