EDGE OPEN PACKING: COMPLEXITY AND COMPUTATIONAL ASPECTS

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Given a graph G, two edges $e_1, e_2 \in E(G)$ are said to have a common edge e if e joins an endvertex of e_1 to an endvertex of e_2. A subset $B \subseteq E(G)$ is an edge open packing (EOP) in G if no two edges of B have a common edge in G, and the maximum cardinality of such a set in G is called the edge open packing number, $\rho_e^o(G)$, of G. In this paper, we prove that the decision version of the EOP number is NP-complete even when restricted to graphs with universal vertices and Eulerian bipartite graphs, respectively. In contrast, we present a linear-time algorithm that computes the parameter for trees. We also solve a problem posed in an earlier paper on this topic. Notably, we characterize the graphs G that attain the upper bound $\rho_e^o(G) \leq |E(G)|/\delta(G)$.

This problem was introduced in [3]. Note that the EOP sets are color classes of the injective edge coloring of graphs ([2]) as well as a generalization of induced matchings ([1, 4]).

1 Main results

We first discuss the following decision problem associated with the EOP number.

Edge open packing problem	
Instance: A graph G and an integer $k \leq |E(G)|$.	(1)
Question: Is $\rho_e^o(G) \geq k$?	

We show that the problem (1) is NP-complete for some special families of graphs.

Theorem 1 Edge Open Packing Problem *is NP-complete even for graphs with universal vertices.*

Moreover, we prove that (1) is in some sense harder than Independent Set Problem as it is known that the independence number of bipartite graphs can be computed in polynomial time.

Theorem 2 Edge Open Packing Problem *is NP-complete even for Eulerian bipartite graphs.*

We prove that the EOP number and an optimal EOP set for any tree T can be computed/constructed in linear time by exhibiting an efficient algorithm for EOP in trees, in which the parameter can be computed in terms of four auxiliary versions of the EOP number which are recursively defined based on rooted trees.

Theorem 3 *There exists a linear-time algorithm for computing the edge open packing number of a tree.*

We define the family $\mathcal{F}$ as follows. Let G be a bipartite graph of minimum degree $k \geq 2$ with partite sets $A \cup C$ and B such that every vertex in B has 1 and $k-1$ neighbors in A and C, respectively.

Theorem 4 *For any graph G of size m, $\rho_e^o(G) = m/\delta(G)$ if and only if either G is a disjoint union of stars or $G \in \mathcal{F}$.*

References

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