WHICH N-VERTEX E-EDGE GRAPH HAS THE MOST H SUBGRAPHS?

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We are interested in finding the graph of given size and order that contains the most copies of a certain small graph as a subgraph. Precisely speaking, for a simple graph H, let $ex(n, e, H)$ denote the maximal number of copies of (not necessarily induced) H subgraphs in a graph with n vertices and e edges. There is no theorem telling us the exact value of $ex(n, e, H)$ or even an asymptotic bound for a general H, but the problem is settled for certain specific graphs.

We will overview the theorem of Ahlswede and Katona about $H = K_{1,2}$, its generalization by Reiher and Wagner about any $H = K_{1,s}$ and the speaker's result concerning the case when H is a 4-edge path. For these graphs, $ex(n, e, H)$ is asymptotically achieved by either a clique (if the edge density is high) or the complement of a clique (if the edge density is low). We also discuss a theorem of Alon that describes infinitely many graphs H for which the clique is always the optimal construction.

Blekherman and Patel proved that for any graph H, $ex(n, e, H)$ is asymptotically achieved by a threshold graph. Gerbner, Patkós, Vizer and the speaker showed that for any H this extremal construction is the clique, provided that the edge density is above a threshold c_H. We also investigate a variant of these problems, when the host graph is bipartite.