STACKED BOOK GRAPHS ARE PERMUTATIONALLY 3-REPRESENTABLE

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A simple graph $G = (V, E)$ is called a word-representable graph if there exists a word w over its vertex set V such that, for all $a, b \in V$, $\overline{ab} \in E$ if and only if a and b alternate in w. The word-representable graphs covers many important classes of graphs including comparability graphs, circle graphs, and 3-colorable graphs. The monograph by Kitaev and Lozin [3] provides a comprehensive account of word-representable graphs, their connections to other contexts, and contributions to the topic.

A word-representable graph is said to be k-word-representable if it is represented by a word in which every letter occurs exactly k times. The smallest k such that a graph is k-word-representable is called the representation number of the graph. If a word representing a word-representable graph is of the form $p_1p_2 \cdots p_k$, where each p_i's is a permutation of its vertices, then the graph is said to be permutationally k-representable. In fact, the class of permutationally representable graphs is precisely the class of comparability graphs [4]. The permutation-representation number (in short *prn*) of a comparability graph is the the minimum value of k such that the graph is permutationally k-representable. It is to be noted that the representation number of a comparability graph is at most its *prn*. The class of complete graphs is precisely the graphs with the *prn* one.

The class of graphs with *prn* at most two is characterized as the class of permutation graphs [1], and the class of circle graphs is characterized as the class of graphs with representation number at most two [2]. In general, it was shown that determining the *prn* and representation number of a permutationally representable graph are computationally hard [5, 2]. In the literature, the *prn* and the representation number for some specific classes of graphs were obtained, in addition to some isolated examples. The classification for the class of graphs with the *prn* at most three is an open problem. In this work, first we reconcile the graphs of with the *prn* at most three. Further, we show that the stacked book graphs have the *prn* as well as the representation number at most three.

References

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