

# STUDY OF THE TOTAL TRIPLE ROMAN DOMINATION IN GRAPHS<sup>1</sup>

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The Total Triple Roman domination in Graphs arises as a new variant of the Roman domination. A Roman domination in graphs is a modeling of a military defensive problem of the Roman empire defined by Cockayne [3] in 2004. Triple Roman domination was introduced by Ahangar et al. [1] in 2021 with the objective of having each territory defended by three legions, minimizing its cost. Let us consider  $f$  as a function  $f : V(G) \rightarrow \{0, 1, 2, 3, 4\}$  in the graph  $G = (V, E)$ , such that,  $f(AN[v]) \geq |AN(v)| + 3$  for any  $v \in V$  with  $f(v) < 3$ , with  $AN(v) \subseteq V$  being the set of adjacent vertices to  $v$  with positive label. Total Triple Roman domination was born as a new variant of Triple Roman domination with the aim of making it more efficient in the face of an individual attack on the nodes. This variant defined by a function  $f$  on the graph  $G$  must satisfy the previous conditions of the Triple Roman domination, in addition to any subgraph induced in  $G$  by the set  $u \in V$ , such that  $f(u) \neq 0$  does not have isolated vertices. The Total Triple Roman domination number  $\gamma_{[t3R]}(G)$  is defined as the minimum of the weight of the sum of the labels  $w(f) = \sum f(v)$  and the function  $f$  defined in  $G$  is a  $\gamma_{[t3R]}(G)$ -function. In this work some bounds are established. Exact values are also studied for some families of graphs such as paths, cycles, bistars, bipartite and spider graphs.

## References

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<sup>1</sup>joint work with Pilar Álvarez-Ruiz, pilar.ruiz@uca.es, J. Carlos Valenzuela-Tripodoro, jcarlos.valenzuela@uca.es

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