DOMINATION IN GRAPHS AND FORBIDDEN CYCLES

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We discuss results showing that if certain cycles are forbidden, then the known upper bounds on core domination parameters can be improved. Let G be a connected graph of order n with minimum degree $\delta(G)$. Let $g(G)$ denote the girth of G, and so $g(G)$ is the length of a shortest cycle in G.

It is known that if $\delta(G) \geq 2$ and $n \geq 8$, then $\gamma(G) \leq \frac{2}{5}n$, where $\gamma(G)$ is the domination number of G. We show that if $\delta(G) \geq 2$ and $n \geq 14$, and if G has no induced 4-cycle and no induced 5-cycle, then $\gamma(G) \leq \frac{3}{8}n$. It is known that if G is a cubic graph, then $\gamma(G) \leq \frac{3}{8}n$. We show that if G is a cubic graph with girth $g(G) \geq 6$ that does not contain a 7-cycle or a 8-cycle, then $\gamma(G) \leq \frac{1}{3}n$.

It is known that if $\delta(G) \geq 2$ and $n \geq 11$, then $\gamma_t(G) \leq \frac{4}{7}n$, where $\gamma_t(G)$ is the total domination number of G. We show that if $n \geq 19$ and G has no induced 6-cycle, then $\gamma_t(G) \leq \frac{6}{11}n$. It is known that if $\delta(G) \geq 3$, then $\gamma_t(G) \leq \frac{1}{2}n$. We show that if $\delta(G) \geq 3$ and G has no induced 6-cycle, then $\gamma_t(G) \leq \frac{4}{9}n$. It is known that if $\delta(G) \geq 4$, then $\gamma_t(G) \leq \frac{3}{7}n$. We show that if $\delta(G) \geq 4$ and G has no 4-cycle, then $\gamma_t(G) \leq \frac{2}{5}n$.

It is known that if $G \neq K_{3,3}$ is a cubic graph, then $i(G) \leq \frac{2}{5}n$, where $i(G)$ is the independent domination number of G. We show that if G is a cubic graph that contain no induced 4-cycle, then $i(G) \leq \frac{3}{8}n$. Furthermore, if G is a bipartite cubic graph that contain no induced 4-cycle, then $i(G) \leq \frac{4}{11}n$.