The 9th Gdańsk Conference on Graph Theory

with panel lessons of summer school

June 16 – 21, 2024

Gdańsk

Organized by

Participants

1. **KARL BARTOLO**
   e-mail: karl.bartolo.16@um.edu.mt
   University of Malta (Malta)

2. **KRZYSZTOF BARTOSZEK**
   e-mail: krzysztof.bartoszek@liu.se
   Linköping University (Sweden)

3. **PAWEŁ BEDNARZ**
   e-mail: pbednarz@prz.edu.pl
   Rzeszów University of Technology (Poland)

4. **ALLAN BICKLE**
   e-mail: aebickle@purdue.edu
   Purdue University (United States)

5. **PETER BORG**
   e-mail: peter.borg@um.edu.mt
   University of Malta (Malta)

6. **PIOTR BOROWIECKI**
   e-mail: p.borowiecki@issi.uz.zgora.pl
   University of Zielona Góra (Poland)

7. **SYLWIA CICHACZ**
   e-mail: cichacz@agh.edu.pl
   AGH University of Science and Technology (Poland)

8. **MACIEJ CISIŃSKI**
   e-mail: cisinski@agh.edu.pl
   AGH University of Science and Technology (Poland)

9. **JOANNA CYMAN**
   e-mail: joanna.cyman@pg.edu.pl
   Gdańsk University of Technology (Poland)

10. **DARIUSZ DERENIOWSKI**
    e-mail: deren@eti.pg.edu.pl
    Gdańsk University of Technology (Poland)

11. **MAGDA DETTLAFF**
e-mail: magda.dettlaff@ug.edu.pl
University of Gdańsk (Poland)

12. **EWA DRGAS-BURCHARDT**
e-mail: e.drgas-burchardt@im.uz.zgora.pl
University of Zielona Góra (Poland)

13. **TINAZ EKIM**
e-mail: tinaz.ekim@bogazici.edu.tr
Boğaziçi University (Turkey)

14. **TITHI DWARY**
e-mail: tithi.dwary@iitg.ac.in
Indian Institute of Technology Guwahati (India)

15. **JOHN STEWART FABILA CARRASCO**
e-mail: john.fabila@ed.ac.uk
University of Edinburgh (United Kingdom)

16. **ANNA FLASZCZYŃSKA**
e-mail: flaszczynska@agh.edu.pl
AGH University of Science and Technology (Poland)

17. **JULIÁN ALBERTO FRESÁN FIGUEROA**
e-mail: jfresan@cua.uam.mx
Universidad Autónoma Metropolitana Unidad Cuajimalpa (Mexico)

18. **HANNA FURMAŃCZYK**
e-mail: hanna.furmanczyk@ug.edu.pl
University of Gdańsk (Poland)

19. **KRZYSZTOF GIARO**
e-mail: giaro@pg.edu.pl
Gdańsk University of Technology (Poland)

20. **JOHN GIMBELL**
e-mail: jggimbel@alaska.edu
University of Alaska (United States)

21. **CARLOS GONZALEZ**
e-mail: carlos.gonzalez@newcastle.ac.uk
Newcastle University (United Kingdom)

22. **ISMAEL GONZÁLEZ YERO**
e-mail: ismael.gonzalez@uca.es
University of Cadiz (Spain)

23. **IZOLDA GORGOL**
e-mail: i.gorgol@pollub.pl
Lublin University of Technology (Poland)

24. **ALEKSANDRA GORZKOWSKA**
e-mail: agorzkow@agh.edu.pl
AGH University of Science and Technology (Poland)

25. **IGOR GRZELEC**
e-mail: grzelec@agh.edu.pl
AGH University of Science and Technology (Poland)

26. **STANISŁAW HALKIEWICZ**
e-mail: smsh@duck.com
Wrocław University of Economics and Business (Poland)

27. **MICHAEL A. HENNING**
e-mail: mahenning@uj.ac.za
University of Johannesburg (Republic of South Africa)

28. **NADZIEJA HODUR**
e-mail: nhodur@agh.edu.pl
AGH University of Science and Technology (Poland)

29. **ANDRZEJ JASTRZĘBSKI**
e-mail: jendrek@eti.pg.edu.pl
Gdańsk University of Technology (Poland)

30. **NAHID YELENE JAVIER NOL**
e-mail: nahid@xanum.uam.mx
Universidad Autonoma Metropolitana Iztapalapa (Mexico)

31. **VILLE JUNNILA**
e-mail: viljun@utu.fi
University of Turku (Finland)

32. **RAFAŁ KALINOWSKI**
e-mail: kalinows@agh.edu.pl
AGH University of Science and Technology (Poland)

33. **ADIL KHAN**
e-mail: thisisadil.ak@gmail.com
IISER Bhopal (India)

34. **SANDI KLAVŽAR**
e-mail: sandi.klavzar@fmf.uni-lj.si
University of Ljubljana (Slovenia)

35. **ADAM KOSTULAK**
e-mail: adam.kostulak@ug.edu.pl
University of Gdańsk (Poland)

36. **ŁUKASZ KOWALIK**
e-mail: lm.kowalik@uw.edu.pl
University of Warsaw (Poland)

37. **JULIA KOZIK**
e-mail: julkozik@agh.edu.pl
AGH University of Science and Technology (Poland)

38. **BARBARA KRUPIŃSKA**
e-mail: bkrupin@agh.edu.pl
AGH University of Science and Technology (Poland)

39. **MAREK KUBALE**
e-mail: kubale@eti.pg.gda.pl
Gdańsk University of Technology (Poland)

40. **ŁUKASZ KUSZNER**
e-mail: lukasz.kuszner@ug.edu.pl
University of Gdańsk (Poland)

41. **DOROTA KUZIAK**
e-mail: dorota.kuziak@uca.es
University of Cadiz (Spain)

42. **ALEKSANDRA LASKOWSKA**
e-mail: alelaskowska@gmail.com
Gdańsk University of Technology (Poland)

43. **MAGDALENA LEMAŃSKA**
e-mail: magdalena.lemanska@pg.edu.pl
Gdańsk University of Technology (Poland)

44. **BERNARDO LLANO**
e-mail: llano@xanum.uam.mx
Universidad Autónoma Metropolitana (Mexico)

45. **LUCAS MADER**
e-mail: maderlucasj@gmail.com
Fort Hays State University (United States)

46. **MARÍA ANTONIA MATEOS CAMACHO**
e-mail: antonia.mateos@uca.es
Universidad de Sevilla (Spain)

47. **ADRIAN MICHALSKI**
e-mail: a.michalski@prz.edu.pl
Rzeszów University of Technology (Poland)

48. **MATEUSZ MIOTK**
e-mail: mateusz.miotk@ug.edu.pl
University of Gdańsk (Poland)

49. **DOOST ALI MOJDEH**
e-mail: damojdeh@umz.ac.ir
University of Mazandaran (Iran)

50. **MERCÈ MORA**
e-mail: merce.mora@upc.edu
Universitat Politècnica de Catalunya (Spain)

51. **KHYODENO MOZHUI**
e-mail: k.mozhui@iitg.ac.in
Indian Institute of Technology Guwahati (India)

52. **KIEKA MYNHARDT**
e-mail: kieka@uvic.ca
University of Victoria (Canada)

53. **DÁNIEL NAGY**
e-mail: nagy.daniel@renyi.hu
HUN-REN Alfréd Rényi Institute of Mathematics (Hungary)

54. **ANNA NENCA**
e-mail: anna.nenca@ug.edu.pl
University of Gdańsk (Poland)

55. **OPEYEMI OYEWUMI**
e-mail: opeyemioluwaoyewumi@gmail.com
Stellenbosch University (South Africa)

56. **IZTOK PETERIN**
e-mail: iztok.peterin@um.si
University of Maribor (Slovenia)

57. **TYTUS PIKIES**
e-mail: tytpikie@pg.edu.pl
Gdańsk University of Technology (Poland)

58. **MONIKA PILŚNIAK**
e-mail: pilsniak@agh.edu.pl
AGH University of Science and Technology (Poland)

59. **MAGDALENA PROROK**
e-mail: prorok@agh.edu.pl
AGH University of Science and Technology (Poland)

60. **JOANNA RACZEK**
e-mail: joanna.raczek@pg.edu.pl
Gdańsk University of Technology (Poland)

61. **STANISŁAW RADZISZOWSKI**
e-mail: spr@cs.rit.edu
Rochester Institute of Technology (United States)

62. **MONIKA ROSICKA**
e-mail: monika.rosicka@ug.edu.pl
University of Gdańsk (Poland)

63. **RIANA ROUX**
e-mail: rianaroux@sun.ac.za
Stellenbosch University (South Africa)

64. **PAWEŁ RZĄŻEWSKI**
e-mail: pawel.rzazewski@pw.edu.pl
Warsaw University of Technology (Poland)

65. **BABAK SAMADI**
e-mail: babak.samadi@imfm.si
University of Ljubljana (Slovenia)

66. **DAYLE SCICLUNA**
e-mail: dayle.scicluna.09@um.edu.mt
University of Malta (Malta)

67. **ELŻBIETA SIDOROWICZ**
e-mail: e.sidorowicz@wmie.uz.zgora.pl
University of Zielona Góra (Poland)

68. **MARIA JOSÉ SOUTO SALORIO**
e-mail: maria.souto.salorio@udc.es
Universidade da Coruña (Spain)

69. **MICHAŁ SZYFELBEIN**
e-mail: s193307@student.pg.edu.pl
Gdańsk University of Technology (Poland)

70. **JING TIAN**
e-mail: jingtian526@126.com
University of Ljubljana (Slovenia), University of Science and Technology (China)

71. **FARAJI TIRAGA**
e-mail: faraji@aims.edu.gh
Stellenbosch University (South Africa)

72. **JERZY TOPP**
e-mail: j.topp@ans-elblag.pl
University of Applied Sciences, Elbląg (Poland)

73. **OMAR TOUT**
e-mail: o.tout@squ.edu.om
Sultan Qaboos University (Oman)

74. **ELŻBIETA TUROWSKA**
e-mail: e.turowska@im.uz.zgora.pl
University of Zielona Góra (Poland)

75. **KRZYSZTOF TUROWSKI**
e-mail: krzysztof.szymon.turowski@gmail.com
Jagiellonian University (Poland)

76. **JAN WOJTKIEWICZ**
e-mail: janwoj@eti.pg.edu.pl
Gdańsk University of Technology (Poland)

77. **IZAJASZ WROSZ**
e-mail: izajasz.wrosz@pg.edu.pl
Gdańsk University of Technology (Poland)

78. **JUAN CARLOS VALENZUELA-TRIPODORO**
e-mail: jcarlos.valenzuela@uca.es
Univesidad de Cadiz (Spain)

79. **RENATA ZAKRZEWSKA**
e-mail: renata.zakrzewska@pg.edu.pl
Gdańsk University of Technology (Poland)

80. **RADOSŁAW ZIEMANN**
e-mail: radoslaw.ziemann@ug.edu.pl
University of Gdańsk (Poland)

81. **RITA ZUAZUA**
e-mail: ritazuazua@ciencias.unam.mx
National Autonomous University of Mexico (Mexico)

82. **ANDRZEJ ŻAK**
e-mail: zakandrz@agh.edu.pl
AGH University of Science and Technology (Poland)

83. **PAWEŁ ŻYLIŃSKI**
e-mail: pawel.zylinski@ug.edu.pl
University of Gdańsk (Poland)

INVITED TALKS

DEFECTIVE RAMSEY NUMBERS: CLASSICAL PROOFS AND COMPUTER ENUMERATIONS

Tinaz Ekim

Boğaziçi University, Turkey

e-mail: tinaz.ekim@bogazici.edu.tr

We investigate a variant of Ramsey numbers called defective Ramsey numbers, introduced by Ekim and Gimbel in 2013, where cliques and independent sets are generalized to k-dense and k-sparse sets, both commonly called k-defective sets. Following some defective parameters in general graphs, we focus on the computation of defective Ramsey numbers in some restricted graph classes: cographs, chordal graphs, bipartite graphs, perfect graphs, split graphs, cacti, and triangle-free graphs. We adopt a two-fold approach to tackle defective Ramsey numbers. We provide direct proofs using structural graph theory. When this technique falls short in obtaining new values of defective Ramsey numbers, we use efficient graph enumeration techniques for structured graphs.

References

[1] T. Ekim, J. Gimbel, Some defective parameters in graphs, Graphs and Combinatorics,Volume 29, Number 2, (2013), 213–224.

[2] Akdemir, T. Ekim, Advances on Defective Parameters in Graphs, Discrete Optimization, 16 (2015), 62–69.

[3] T. Ekim, J. Gimbel, O. Seker, Small 1-Defective Ramsey Numbers in Perfect Graphs, Discrete Optimization, 34 (2019) 100548.

[4] Y.E. Demirci, T. Ekim, J. Gimbel, M.A. Yildiz, Exact Values of Defective Ramsey Numbers in Graph Classes, Discrete Optimization, 42 (2021) 100673.

[5] Y.E. Demirci, T. Ekim, M.A. Yildiz, Defective Ramsey Numbers and Defective Cocolorings in Some Subclasses of Perfect Graphs, Graphs and Combinatorics, (2023) 39:18.

[6] T. Ekim, B. Erdem, J. Gimbel, Sparse Sets in Triangle-free Graphs, submitted.

DISJOINT COPIES OF GRAPHS IN EXTREMAL GRAPH THEORY

Izolda Gorgol

Lublin University of Technology

e-mail: i.gorgol@pollub.pl

Most of graph theory problems were studied firstly for connected graphs. In the talk I will present selected results which involves disconnected ones with the special focus on graphs consisting of disjoint copies of a certain connected graph. These results are connected with widely understood extremal graph theory that explores the extremal (maximum or minimum) properties of graphs subject to certain constraints. The issues I've selected will be Ramsey, Induced Ramsey and Turán numbers.

DOMINATION IN GRAPHS AND FORBIDDEN CYCLES

Michael A. Henning

University of Johannesburg

e-mail: mahenning@uj.ac.za

We discuss results showing that if certain cycles are forbidden, then the known upper bounds on core domination parameters can be improved. Let G be a connected graph of order n with minimum degree $\delta(G)$. Let $g(G)$ denote the girth of G, and so $g(G)$ is the length of a shortest cycle in G.

It is known that if $\delta(G) \geq 2$ and $n \geq 8$, then $\gamma(G) \leq \frac{2}{5}n$, where $\gamma(G)$ is the domination number of G. We show that if $\delta(G) \geq 2$ and $n \geq 14$, and if G has no induced 4-cycle and no induced 5-cycle, then $\gamma(G) \leq \frac{3}{8}n$. It is known that if G is a cubic graph, then $\gamma(G) \leq \frac{3}{8}n$. We show that if G is a cubic graph with girth $g(G) \geq 6$ that does not contain a 7-cycle or a 8-cycle, then $\gamma(G) \leq \frac{1}{3}n$.

It is known that if $\delta(G) \geq 2$ and $n \geq 11$, then $\gamma_t(G) \leq \frac{4}{7}n$, where $\gamma_t(G)$ is the total domination number of G. We show that if $n \geq 19$ and G has no induced 6-cycle, then $\gamma_t(G) \leq \frac{6}{11}n$. It is known that if $\delta(G) \geq 3$, then $\gamma_t(G) \leq \frac{1}{2}n$. We show that if $\delta(G) \geq 3$ and G has no induced 6-cycle, then $\gamma_t(G) \leq \frac{4}{9}n$. It is known that if $\delta(G) \geq 4$, then $\gamma_t(G) \leq \frac{3}{7}n$. We show that if $\delta(G) \geq 4$ and G has no 4-cycle, then $\gamma_t(G) \leq \frac{2}{5}n$.

It is known that if $G \neq K_{3,3}$ is a cubic graph, then $i(G) \leq \frac{2}{5}n$, where $i(G)$ is the independent domination number of G. We show that if G is a cubic graph that contain no induced 4-cycle, then $i(G) \leq \frac{3}{8}n$. Furthermore, if G is a bipartite cubic graph that contain no induced 4-cycle, then $i(G) \leq \frac{4}{11}n$.

ETERNAL EVICTION AND INDEPENDENCE[1]

Kieka Mynhardt

University of Victoria

e-mail: kieka@uvic.ca

Graph protection involves the deployment of mobile guards on the vertices of a graph. The various protection models can be described as two-player games, alternating between a defender and an attacker: the defender chooses the original positions of the guards, as well as the responses to the attacker, and the attacker chooses the locations of the attacks; we say the attacker attacks the vertices. In the (eternal) eviction game, at most one guard is located at each vertex, and each configuration of guards is a dominating set of the graph. The attacker attacks a vertex occupied by a guard, provided this vertex has at least one unoccupied neighbour. The defender moves the guard to an unoccupied neighbour; only one guard is allowed to move in response to an attack. The defender wins the game if they can successfully defend any sequence of attacks, including sequences that are infinitely long; the attacker wins otherwise. In other words, the attacker's goal is to force the defender into a configuration of guards that is not dominating. The smallest number of guards that can defend a graph G against any sequence of attacks is called the eviction number of G, denoted by $e^{\infty}(G)$. The eviction game was introduced by Klostermeyer, Lawrence, and MacGillivray in 2016.

In this presentation I will demonstrate that the eviction number behaves different from other domination parameters. This anomaly causes problems when we try to prove results for $e^{\infty}(G)$. I will illustrate this by discussing a proof of an upper bound for $e^{\infty}(G)$ in terms of $\alpha(G)$, the independence number of G.

[1]Joint work with Gary MacGillivray and Virgélot Virgile

RECENT PROGRESS ON SMALL AND LARGE RAMSEY NUMBERS

Stanisław Radziszowski

Rochester Institute of Technology

e-mail: spr@cs.rit.edu

The new revision #17 of the dynamic survey *Small Ramsey Numbers* at the *Electronic Journal of Combinatorics* has been just completed. In this talk we will overview new developments since 2021 reported therein: there were breakthrough in asymptotics, some amazing improvements of the bounds on the classical Ramsey numbers, several less known but also very impressive results on general graph Ramsey numbers, and a large number of contributions across the area. We will also reveal some interesting details of the logistics of evolving survey and its special features.

TUTORIALS

VISIBILITY CONCEPTS IN GRAPH THEORY

Sandi Klavžar

University of Ljubljana, Slovenia
Institute of Mathematics, Physics and Mechanics, Slovenia

e-mail: sandi.klavzar@fmf.uni-lj.si

Given a connected graph G and a set of vertices $X \subseteq V(G)$, two vertices $x, y \in V(G)$ are called to be X-*visible* if there is a shortest x, y-path (also called geodesic) whose interior vertices do not belong to X. Then X is

- a *mutual-visibility set*: if any two vertices of X are X-visible;
- an *outer mutual-visibility set*: if any two vertices $x, y \in X$ and any two vertices $x \in X$ and $y \in \overline{X}$ are X-visible;
- a *dual mutual-visibility set*: if any two vertices $x, y \in X$ and any two vertices $x, y \in \overline{X}$ are X-visible; and
- a *total mutual-visibility set*: if any two vertices $x, y \in V(G)$ are X-visible.

In this tutoring, we will present fundamental results on these concepts. A special attention will be given to graphs of diameter two as there unexpected connections with some classical mathematical problems and concepts arise.

ALGEBRAIC TECHNIQUES IN PARAMETERIZED GRAPH ALGORITHMS

ŁUKASZ KOWALIK

University of Warsaw

e-mail: lm.kowalik@uw.edu.pl

For a number of algorithmic graph problems that are NP-hard, it is possible to get considerable speed-ups by phrasing the task as a kind of counting problem and then using algebraic techniques. We will see a number of such examples, including:

- Hamiltonian cycle in $2^n poly(n)$ time using inclusion-exclusion principle,
- Vertex coloring in time $2^n poly(n)$ time using cover product or fast subset convolution,
- Finding a k-vertex path in $2^k poly(n)$ time using polynomials over a finite field.

The material will be mostly based on Chapter 11 of the textbook Parameterized Algorithms by Cygan, Fomin, Kowalik, Lokshtanov, Marx, Pilipczuk, Pilipczuk, Saurabh.

H-FREE GRAPHS: FROM STRUCTURE TO ALGORITHMS

Paweł Rzążewski

Warsaw University of Technology

e-mail: pawel.rzazewski@pw.edu.pl

One of the active areas of algorithmic graph theory is to investigate how the restrictions imposed on the set of input instances influence the complexity of computational problems. Quite often we can witness an interesting interplay between graph-theoretic and algorithmic results: a good understanding on the structure of instances may help in the design of efficient algorithms.

During the tutorial we will show some tools and techniques that can be used to develop algorithms for graphs that exclude a fixed graph F as an induced subgraph. We will mostly focus on the case that F is a path.

POSTERS

MAJORITY COLORING OF GRAPHS: THEORETICAL INSIGHTS AND PRACTICAL APPLICATION

Aleksandra Laskowska

Gdańsk University of Technology

e-mail: alelaskowska@gmail.com

Let $G = (V, E)$ be a simple, undirected graph and map $c : V \to C$ a coloring, where C is a set of colors. **Majority coloring** is such coloring that every $v \in V$ has at most $\frac{1}{2}deg(v)$ neighbours coloured $c(v)$. In this poster I show significant definitions and theorems regarding vertex majority coloring. Additionally, putting this theoretical concept into practice is discussed.

CERTIFIED DOMINATION IN GRAPHS USING BINARY LINEAR PROGRAMMING

Mateusz Miotk, Joanna Raczek

University of Gdańsk, Poland, Gdańsk University of Technology, Poland

e-mail: mateusz.miotk@ug.edu.pl, joanna.raczek@pg.edu.pl

A set D of vertices of a graph $G = (V_G, E_G)$ is a *dominating set* of G if every vertex in $V_G - D$ is adjacent to at least one vertex in D. The *domination number* of a graph G, denoted by $\gamma(G)$, is the cardinality of a smallest dominating set of G. A subset $D \subseteq V_G$ is called a *certified dominating set* of G if D is a dominating set of G, and every vertex in D has either zero or at least two neighbours in $V_G - D$. The cardinality of a smallest certified dominating set of G is called the *certified domination number* of G, and it is denoted by $\gamma_{\text{cer}}(G)$.

A BLP (binary linear program) is constructed to derive the system of linear constraints corresponding to the certified domination conditions. The objective is to drive the minimum cardinality of the certified dominating set problem through a linear optimisation problem. This approach is used to identify the optimal domination set in different categories of graphs.

The clarity of the results demonstrates that the BLP algorithm is effective in recognising the minimum certified dominating set associated with the certified domination set. This will result in significant advances in theory, practice, research, and applications.

References

[1] M. Dettlaff, M. Lemańska, M. Miotk, J. Topp, R. Ziemann, and P. Żyliński, Graphs with equal domination and certified domination numbers, Opuscula Math. 39 (2019), no. 6, 815–827.

[2] M. Dettlaff, M. Lemańska, J. Topp, R. Ziemann, and P. Żyliński, Certified domination, AKCE International Journal of Graphs and Combinatorics (2018).

[3] M. Miotk, J. Raczek, Modelling Efficient Fire Safety Water Networks by Certified Domination, submitted.

CERTIFIED DOMINATION IN WATER SUPPLY NETWORKS FOR FIRE SAFETY

Mateusz Miotk, Joanna Raczek

University of Gdańsk, Poland, Gdańsk University of Technology, Poland

e-mail: mateusz.miotk@ug.edu.pl, joanna.raczek@pg.edu.pl

Providing water to the fire protection water supply network is a crucial aspect of the overall fire protection and life safety strategy of an entire community. Currently, as new buildings are emerging, necessary calculations are being performed so that the buildings are complied with fire safety regulations.

Before everything it is important to make sure that the proper amount of water is available to the responding fire department for both suppression of the fire in the building, and protection of any exposed buildings. All water-based fire protection systems need water. Without access to an adequate water supply these systems will not function properly.

We introduce a theoretical model of a water supply network given in the language of graph theory. The model uses the certified dominating sets to focus on placing the water supply issues and hence other, less important parameters are omitted.

A set D of vertices of a graph $G = (V, E)$ is a *dominating set* of G if every vertex in $V - D$ is adjacent to at least one vertex in D. The *domination number* of a graph G, denoted by $\gamma(G)$, is the cardinality of a smallest dominating set of G. A subset $D \subseteq V$ is called a *certified dominating set* of G if D is a dominating set of G, and every vertex in D has either zero or at least two neighbours in $V - D$. The cardinality of a smallest certified dominating set of G is called the *certified domination number* of G, and it is denoted by $\gamma_{\text{cer}}(G)$.

Thanks to the minimum certified dominating sets it is possible to determine where in the environment to place pumping stations and wells to meet, fire safety requirements, while minimising the cost. We assume the cost of installing a pumping station to be approximately 2.5 times that of a well. The objective is to ensure that every location without a water source is connected by a pipe to a pumping station, thereby ensuring that the water pressure requirements of user locations are met. Also, a place with a well should be connected only to places with a well or a pumping station on order to avoid any pressure decreases.

The aforementioned approach would be further reinforced by the presentation of case studies in which cost savings are demonstrated and, simultaneously, compliance with relevant fire safety standards is supported in a different context. Consequently, the new approach must act once more as a practical tool for urban planners and engineers in promoting a systemic approach to improvements in fire safety infrastructure.

References

[1] M. Dettlaff, M. Lemańska, M. Miotk, J. Topp, R. Ziemann, and P. Żyliński, Graphs with equal domination and certified domination numbers, Opuscula Math. 39 (2019), no. 6, 815–827.

[2] M. Dettlaff, M. Lemańska, J. Topp, R. Ziemann, and P. Żyliński, Certified domination, AKCE International Journal of Graphs and Combinatorics (2018).

[3] M. Miotk, J. Raczek, Modelling Efficient Fire Safety Water Networks by Certified Domination, submitted.

INTERACTIVE SEARCH IN GRAPHS

Izajasz Wrosz

Gdańsk University of Technology

e-mail: izajasz.wrosz@pg.edu.pl

Searching plays a fundamental role in computer science and computer engineering due to its ubiquitous real-world applications and its numerous connections to other important computational problems. In searching we want to locate a known element, whose location in the search space is unknown, by querying different locations of the search space in a sequence of steps. In interactive search an emphasis is made on the type and amount of information revealed through the queries, and how to exploit this information in search algorithms. In this poster, we describe applications of an interactive search model (i.e., binary search in node-weighted trees) in data retrieval systems. In this search model, in each step, the algorithm queries a vertex q and receives an answer, that either q is the desired element, or receives the neighbor of q closer to the target than q. While each query has a cost given by the weight function, the goal is to find an adaptive search strategy requiring the minimum cost in the worst case.

CONTRIBUTED TALKS

SOME HARMONIC NUMBER IDENTITIES FOR PHYLOGENETIC TREE ANALYSIS

Krzysztof Bartoszek

Linköping University

e-mail: krzysztof.bartoszek@liu.se, krzbar@protonmail.ch

Values associated with phylogenetic trees like the total tree area [4] or the cophenetic index [5] can be represented through the height of the tree, and the time to coalescent of a random pair of tips. In this way the given index, for a random tree, can be studied by considering a pair of (dependent) one–dimensional random variables. Control over their moments will immediately provide information on the behaviour of these indices [1, 3, 6]. In order to obtain these moments, for the pure birth tree, one has to consider rather involved harmonic and quadratic harmonic sums. In the finite term case, these sums often turn out to have closed form formulæ in terms of harmonic numbers. However, surprisingly, symbolic algebra systems do not seem to be able (at least out of the box) to find these final forms. In our talk we will show how these sums arrive in the analysis of tree heights, in what situations computer algebra systems fail, and how one can approach these sums [2].

References

[1] K. Bartoszek, Quantifying the effects of anagenetic and cladogenetic evolution. Mathematical Biosciences 254 (2014) 42–57.

[2] K. Bartoszek, Closed and asymptotic formulæ for harmonic and quadratic harmonic sums. ArXiv e-prints (2023). arXiv: 2312.15366.

[3] K. Bartoszek, Exact and approximate limit behaviour of the Yule tree's cophenetic index. Mathematical Biosciences 303 (2018), 26–45.

[4] A. Mir, F. Rosselló, The mean value of the squared path–difference distance for rooted phylogenetic trees. Journal of Mathematical Analysis and Applications 371 (2010), 168–176.

[5] A. Mir, F. Rosselló and L. Rotger, A new balance index for phylogenetic trees. Mathematical Biosciences 241 (2013), 125–136.

[6] S. Sagitov, K. Bartoszek, Interspecies correlation for neutrally evolving traits. Journal of Theoretical Biology. 309 (2012) 11–19.

ON VARIOUS TYPES OF PROPER SECONDARY DOMINATING SETS

Paweł Bednarz, Adrian Michalski and Mateusz Pirga

Rzeszow University of Technology

e-mail: pbednarz@prz.edu.pl, a.michalski@prz.edu.pl, m.pirga@prz.edu.pl

Let $k \geq 1$ be an integer. A subset $D \subset V(G)$ is (1,k)-dominating if for every vertex $v \in V(G) \setminus D$ there are $u, w \in D$ such that $uv \in E(G)$ and $d_G(v, w) \leq k$. If $k = 1$ then we obtain the definition of (1,1)-dominating sets, which are also known as 2-dominating sets. If $k = 2$ then we have the concept of (1,2)-dominating sets, see [1].

In [2] Michalski et. al introduced the concept of proper (1,2)-dominating sets to distinguish (1,2)-dominating sets from (1,1)-dominating sets. A *proper* (1,2)-*dominating set* is a (1,2)-dominating set that is not (1,1)-dominating. Basing on this idea, Bednarz and Pirga in [3] defined proper 2-dominating sets i.e. 2-dominating sets which are not 3-dominating.

In this talk we present some results concerning proper (1,2)-dominating sets and proper 2-dominating sets, in particular we focus on the problem of their existence. Moreover, we show relations between parameters of these types of domination.

References

[1] S.M. Hedetniemi, S.T. Hedetniemi, J. Knisely, D.F. Rall, *Secondary domination in graphs*, AKCE International Journal of Graphs and Combinatorics, 5, (2008), 103–115.

[2] A. Michalski, I. Włoch, M. Dettlaff, M. Lemańska, *On proper (1,2)-dominating sets*, Mathematical Methods in the Applied Sciences, 45(11), (2022), 7050–7057.

[3] P. Bednarz, M. Pirga, *On proper 2-dominating sets in graphs*, Symmetry, 16, (2024), 296.

PLANE TRIANGULATIONS WITHOUT SPANNING 2-TREES

<u>ALLAN BICKLE</u>

Purdue University

e-mail: aebickle@purdue.edu

GUNNAR BRINKMANN

Ghent University

e-mail: Gunnar.Brinkmann@UGent.bel

A 2-tree is a graph that can be formed by starting with a triangle and iterating the operation of making a new vertex adjacent to two adjacent vertices of the existing graph. Leizhen Cai asked in 1995 whether every maximal planar graph contains a spanning 2-tree. We answer this question in the negative by constructing an infinite class of maximal planar graphs that have no spanning 2-tree. We also show that the largest spanning tree may have an arbitrarily small fraction of all vertices and find some criteria that guarantee a spanning 2-tree.

References

[1] L. Cai, Spanning 2-trees. In: Kanchanasut K., Levy JJ. (eds) Algorithms, Concurrency and Knowledge. ACSC 1995. Lecture Notes in Computer Science, vol 1023. Springer, Berlin, Heidelberg, (1995).

[2] L. Cai, On spanning 2-trees in a graph. Discrete Appl. Math. 74 (1997), 203—216.

ISOLATION OF GRAPHS

PETER BORG

University of Malta

e-mail: peter.borg@um.edu.mt

Given a set $\mathcal{F}$ of graphs, we call a copy of a graph in $\mathcal{F}$ an $\mathcal{F}$-graph. The $\mathcal{F}$-isolation number of a graph G, denoted by $\iota(G, \mathcal{F})$, is the size of a smallest subset D of the vertex set $V(G)$ such that the closed neighbourhood $N[D]$ of D intersects the vertex sets of the $\mathcal{F}$-graphs contained by G (equivalently, $G - N[D]$ contains no $\mathcal{F}$-graph). When $\mathcal{F}$ consists of a 1-clique, $\iota(G, \mathcal{F})$ is the domination number of G. When $\mathcal{F}$ consists of a 2-clique, $\iota(G, \mathcal{F})$ is the vertex-edge domination number of G. The general $\mathcal{F}$-isolation problem was introduced by Caro and Hansberg [10] in 2017. They established many results on $\mathcal{F}$-isolation numbers and posed several problems. Solutions will be presented together with most of the isolation results to date.

References

[1] P. Borg, Isolation of cycles, Graphs and Combinatorics 36 (2020), 631–637.

[2] P. Borg, K. Fenech and P. Kaemawichanurat, Isolation of k-cliques, Discrete Mathematics 343 (2020), paper 111879.

[3] P. Borg, K. Fenech and P. Kaemawichanurat, Isolation of k-cliques II, Discrete Mathematics 345 (2022), paper 112641.

[4] P. Borg and P. Kaemawichanurat, Extensions of the Art Gallery Theorem, Annals of Combinatorics 27 (2023), 31–50.

[5] P. Borg, Isolation of connected graphs, Discrete Applied Mathematics 339 (2023), 154–165.

[6] P. Borg, Isolation of regular graphs, stars and k-chromatic graphs, arXiv:2303.13709 [math.CO].

[7] P. Borg, Isolation of regular graphs and k-chromatic graphs, arXiv:2304.10659 [math.CO].

[8] P. Borg, K. Bartolo and D. Scicluna, Isolation of squares in graphs, arXiv:2310.09128 [math.CO].

[9] G. Boyer and W. Goddard, Disjoint isolating sets and graphs with maximum isolation number, arXiv:2401.03933 [math.CO].

[10] Y. Caro and A. Hansberg, Partial domination - the isolation number of a graph, FiloMath 31:12 (2017), 3925–3944.

[11] M. Lemańska, M. Mora and M.J. Souto–Salorio, Graphs with isolation number equal to one third of the order, Discrete Mathematics 347 (2024), paper 113903.

[12] G. Zhang and B. Wu, $K_{1,2}$-isolation in graphs, Discrete Applied Mathematics 304 (2021), 365–374.

[13] P. Żyliński, Vertex-edge domination in graphs, Aequationes Mathematicae 93 (2019), 735–742.

COMPUTATIONAL COMPLEXITY OF GREEDY PARTITIONING OF GRAPHS[2]

Piotr Borowiecki

University of Zielona Góra

e-mail: p.borowiecki@issi.uz.zgora.pl

In this talk we consider a variant of graph partitioning problem consisting in partitioning the vertex set into the minimum number of sets such that each of them induces a graph in a fixed hereditary class of graphs (property). For various properties we will discuss the computational complexity of several problems arising when partitions are generated by the greedy algorithm. In this context, we will point out the cases that are computationally hard, and those that can be solved in polynomial time. We will also present a lower bound based on the Exponential-Time Hypothesis as well as some basic result on generalized independence and domination allowing the dynamic programming approach in the construction of an exact algorithm. We will also mention an application of the above concepts to the construction of new χ-bounded classes of graphs.

References

[1] E. Bonnet, F. Foucaud, E. J. Kim, F. Sikora, Complexity of Grundy colorings and its variants. *Discrete Applied Mathematics* 243 (2018) 99–114.

[2] P. Borowiecki, Computational aspects of greedy partitioning of graphs. *Journal of Combinatorial Optimization* 35(2) (2018) 641–665.

[3] R. Belmonte, E. J. Kim, M. Lampis, V. Mitsou, Y. Otachi, Grundy distinguishes treewidth from pathwidth, 28th *Annual European Symposium on Algorithms* (ESA 2020), LIPIcs 173 (2020) 14:1–14:19.

[2]Results partially obtained in cooperation with Ela Sidorowicz, Darek Dereniowski and Ewa Drgas-Burchardt

ON A-CORDIAL CATERPILLARS[3]

Sylwia Cichacz

AGH University of Kraków

e-mail: cichacz@agh.edu.pl

Hovey introduced A-cordial labelings as a generalization of cordial and harmonious labelings [3]. If A is an Abelian group, then a labeling $f\colon V(G) \to A$ of the vertices of some graph G induces an edge labeling on G; the edge uv receives the label $f(u) + f(v)$. A graph G is A-cordial if there is a vertex-labeling such that (1) the vertex label classes differ in size by at most one and (2) the induced edge label classes differ in size by at most one.

In the literature, mostly cordial labeling in cyclic groups is studied. There is a famous (still open) conjecture which states that all trees are $\mathbb{Z}_k$-cordial for all k [3]. The situation changes a lot if A is not cyclic. It was proved that all trees, except P_4 and P_5, are $\mathbb{Z}_2^2$-cordial [1].

Patrias and Pechenik posed a conjecture that for every group A there is an A-cordial labeling for almost every path [4]. Erickson et al. extended the conjecture for all trees [1].

In the talk, we show that the conjecture holds for paths [2] but it is not true for general trees - even if we consider an A-rainbow coloring instead of A-cordial (i.e. an A-cordial labeling in which $|A| = |V(G)|$) of caterpillars. Moreover, we will show some correspondence of A-cordial caterpillars and Cayley digraphs on A.

References

[1] W.Q. Erickson, D. Herden, J. Meddaugh, M.R. Sepanski, I. Echols, C. Hammon, J. Marchena-Menendez, J. Mohn, B. Radillo-Murguia, I. Ruiz-Bolanos, *Klein cordial trees and odd cyclic cordial friendship graphs*, Discrete Math., 346(9) (2023) 113488.

[2] S. Cichacz, *On some graph-cordial Abelian groups*, Discrete Math. 345 (2022) 112815.

[3] M. Hovey, *A-cordial graphs*, Discrete Math., 93 (1991) 183–194.

[4] R. Patrias, O. Pechenik, *Path-cordial abelian groups*, Austral. J. Comb. 80(1) (2021) 157–166.

[3]joint work with Barbara Krupińska and Mariusz Woźniak

TREE PACKING CONJECTURE

Maciej Cisiński and Andrzej Żak

AGH University of Kraków

e-mail: cisinski@agh.edu.pl, zakandrz@agh.edu.pl

The Tree Packing Conjecture (TPC) by Gyárfás states that any set of trees $T_2, \ldots, T_{n-1}, T_n$ such that T_i has i vertices pack into K_n. The conjecture is true for bounded degree trees, but in general, it is widely open. Bollobás proposed a weakening of TPC which states that k largest trees pack. We prove, among others, that seven largest trees pack.

References

[1] A. Gyárfás and J. Lehel, Packing trees of different order into K_n, Combinatorics (Proc. Fifth Hungarian Colloq., Keszthely, 1976), Colloq. Math. Soc. János Bolyai, vol. 18, North-Holland, Amsterdam, 1978, 463–469.

[2] A. Żak, Packing large trees of consecutive orders, Discrete Math. 340(2) (2017) 252–263.

INTERVAL COLOURING THICKNESS VIA THE ERDŐS FAMILY OF GRAPHS

Ewa Drgas-Burchardt

University of Zielona Góra

e-mail: e.drgas-burchardt@im.uz.zgora.pl

Let $\theta_{int}(G)$ denote the minimum number of parts in a partition of the edge set of the graph G such that graphs induced by all the parts are interval colourable. Giving a finite projective plane $\pi(n)$ of order n with the sets W,L of points and lines, respectively, $Erd(n)$ is known to be a graph with vertex set $W \cup L \cup \{u\}$ and edge set $\{wl :\ w \in W, l \in L$ and w is incident to $l\ \} \cup \{ul : l \in L\}$. Next, if $L = \{l_1, \ldots, l_{n^2+n+1}\}$ and a sequence $r_1, \ldots, r_{n^2+n+1}$ of positive integers is given, by $Erd(r_1, \ldots, r_{n^2+n+1})$ we mean a graph resulting from $Erd(n)$ by a multiplication of the vertex l_i with r_i vertices made for all $i \in [n^2+n+1]$. Graphs $Erd(r_1, \ldots, r_{n^2+n+1})$ constructed for all possible finite projective planes and all possible parameters $r_1, \ldots, r_{n^2+n+1}$ form the Erdős family of graphs.

Let l be a fixed line of a finite projective plane $\pi(n)$ and $w_1, \ldots, w_{n+1}$ be all points incident to l. Next let for $i \in [n+1]$, a set L_i consist of all lines incident to w_i that are different from l. We prove that if an ordering $l_1, \ldots, l_{n^2+n+1}$ of the set L is given and positive integers $r_1, \ldots, r_{n^2+n+1}$ are such that at least t different indices i from $[n+1]$ satisfy $r_k = r_j$ if $l_k, l_j \in L_i$, then

$$\theta_{int}(Erd(r_1, \ldots, r_{n^2+n+1})) \leq \max\Big\{2, \Big\lceil \frac{n+2-t}{2} \Big\rceil \Big\}.$$

Consequently, a tight upper bound of $\left\lceil \frac{n+2}{2} \right\rceil$ on $\theta_{int}(Erd(r_1, \ldots, r_{n^2+n+1}))$ is valid in the whole Erdős family of graphs.

References

[1] A.S. Asratian, C.J. Casselgren, P.A. Petrosyan, Decomposing graphs into interval colorable subgraphs and no-wait multi-stage schedules, Discrete Appl. Math. 335 (2023) 25–35, doi:10.1016/j.dam.2022.07.015.

[2] A.A. Albert, R. Sandler (1968), An Introduction to Finite Projective Planes, New York: Holt, Rinehart and Winston.

ON OPERATIONS PRESERVING WORD-REPRESENTABILITY OF GRAPHS

TITHI DWARY AND K. V. KRISHNA

Indian Institute of Technology Guwahati, India

e-mail: tithi.dwary@iitg.ac.in, kvk@iitg.ac.in

A simple graph is called a word-representable graph if there is a word over its vertex set such that any two vertices are adjacent in the graph if and only if they alternate in the word. The class of word-representable graphs was first studied by Sergey Kitaev and Steven Seif in the context of Perkin semigroup [4]. Over the years an extensive literature has developed on this topic, impacting various fields of mathematics and computer science. A word-representable graph is a k-word-representable graph, if it is represented by a word in which every letter appears exactly k times. The smallest k such that a graph is k-word-representable is said to be the representation number of the graph. A word-representable graph is said to be a permutationally representable graph, if it can be represented by a word that is a concatenation of permutations on their vertices. The class of comparability graphs, graphs which admit transitive orientations, is precisely the class of permutationally representable graphs [4]. The smallest k such that a permutationally representable graph is represented by a concatenation of k permutations on its vertices is called the permutation-representation number (in short, *prn*) of the graph. Further, it is known that the general problems of determining the *prn* of a permutationally representable graph, and the representation number of a word-representable graph are computationally hard. For a detailed introduction to this topic, one may refer to the monograph [3].

The graph operations were proved to be useful for determining the representation number of graphs. For example, it was proved in [5] that 3-subdivision of every graph is 3-word-representable and utilizing this, the representation number of prism is determined. It was proved in [3] that the class of word-representable graphs is closed under certain graph operations such as connecting two graphs by an edge, and gluing two graphs at a vertex. Moreover, the representation numbers of the resulting graphs were obtained. Some fundamental graph operations viz., edge-deletion, edge addition do not necessarily preserve the word-representability; however, certain sufficient conditions on graphs to preserve word-representability with respect to these operations were established [1].

In this work, we obtain necessary and sufficient conditions for permutation representability of graphs with respect to the following operations: gluing two graphs at a vertex, replacing a vertex by a module, and lexicographical product of graphs. Further, we obtain the *prn*s of the resultant graphs. A modular decomposition of a graph is a partition of the vertex set of the graph into modules. While it was introduced to study the structure of comparability graphs, it has applications in the theory of posets, and scheduling problems. In this work, we extend the characterization of comparability graphs with respect to the modular decomposition (given in [6]) to word-representable graphs. Accordingly, we determine the representation number of a word-representable graph in terms of the *prn*s of its modules and the representation number of the quotient graph. In this connection, we also obtain a complete answer to the open problem posed by Kitaev and Lozin [3, Chapter 7] on the word-representability of the lexicographical product of graphs.

References

[1] I. Choi, J. Kim and M. Kim, On operations preserving semi-transitive orientability of graphs. J. Comb. Optim. 37 (2019), 1351–1366.

[2] M. M. Halldórsson, S. Kitaev and A. Pyatkin, Semi-transitive orientations and word-representable graphs. Discrete Appl. Math. 201 (2016), 164–171.

[3] S. Kitaev and and V. Lozin, Words and graphs. Monographs in Theoretical Computer Science. An EATCS Series. Springer, Cham, (2015).

[4] S. Kitaev and S. Seif, Word problem of the Perkin semigroup via directed acyclic graphs. Order 25(3) (2008), 177–194.

[5] S. Kitaev and A. Pyatkin, On representable graphs. J. Autom. Lang. Comb. 13(1) (2008), 45–54.

[6] M. C. Golumbic, Algorithmic graph theory and perfect graphs. Annals of Discrete Mathematics. Elsevier Science B.V., Amsterdam, second edition, 57 (2004).

DISTINGUISHING VERTICES OF GRAPHS USING SEQUENCES

Anna Flaszczyńska

AGH University of Kraków

e-mail: flaszczynska@agh.edu.pl

In the paper [1] the authors distinguish vertices of a graph by sequences. This talk is about distinguishing vertices of a hypercube by sequences. Let f be the edge coloring of an n-dimensional hypercube. In a hypercube, we can define the order of edges, which results from the structure of this graph. Next, we can assign a sequence of colors to each vertex in such a way that the i-th element of this sequence is the color of the i-th edge coming from this vertex. We want to find a minimum number of colors to distinguish each pair of vertices in an n-dimensional hypercube.

References

[1] B. Seamone and B. Stevens, *Sequence variations of the 1-2-3 Conjecture and irregularity strength*, Discrete Mathematics and Theoretical Computer Science 15(1) (2013), 15-–28.

THE BIPLANAR TREE GRAPH

JULIÁN ALBERTO FRESÁN-FIGUEROA

Universidad Autónoma Metropolitana Unidad Cuajimalpa

e-mail: jfresan@cua.uam.mx

ANA PAULINA FIGUEROA

Instituto Tecnológico Autónomo de México

e-mail: apaulinafg@gmail.com

The complete twisted graph of order n, denoted by T_n, is a complete simple topological graph with vertices $u_1, u_2, \ldots, u_n$, where two edges u_iu_j and $u_{i'}u_{j'}$ intersect if and only if $i < i' < j' < j$ or $i' < i < j < j'$. The convex geometric complete graph of order n, denoted by G_n, is a convex geometric graph with vertices $v_1, v_2, \ldots, v_n$ arranged counterclockwise, with each pair of vertices being adjacent. A biplanar tree of order n is a labeled tree with vertex set $\{v_1, v_2, \ldots, v_n\}$ that can be embedded in both T_n and G_n as a planar graph. Given a connected graph G, the (combinatorial) tree graph $\mathcal{T}(G)$ is a graph whose vertices are the spanning trees of G, and two trees P and Q are adjacent in $\mathcal{T}(G)$ if there exist edges $e \in P$ and $f \in Q$ such that $Q = P - e + f$. For all positive integers n, $\mathcal{T}(n)$ denotes the graph $\mathcal{T}(K_n)$. The biplanar tree graph, $\mathcal{B}(n)$, is the subgraph of $\mathcal{T}(n)$ induced by the biplanar trees of order n. In this conference, we characterize biplanar trees and talk about some properties and structure of the biplanar tree graph.

References

[1] A. P. Figueroa, J. Fresán-Figueroa, The biplanar tree graph. Boletín de la Sociedad Matemática Mexicana, 26(3) (2020), 795–806.

[2] J. Pach, J. Solymosi, and G. Tóth, Unavoidable configurations in complete topological graphs. Discrete and Computational Geometry 30 (2003): 311–320.

[3] E. Omaña-Pulido, and E. Rivera-Campo. "Notes on the twisted graph." Computational Geometry: XIV Spanish Meeting on Computational Geometry, EGC 2011, Dedicated to Ferran Hurtado on the Occasion of His 60th Birthday, Alcalá de Henares, Spain, June 27–30, 2011, Revised Selected Papers. Springer Berlin Heidelberg, 2012.

THE MOBILE MUTUAL-VISIBILITY PROBLEM IN GRAPHS

Magda Dettlaff

University of Gdańsk, Poland

e-mail: magda.dettlaff@ug.edu.pl

Magdalena Lemańska

Gdańsk University of Technology, Poland

e-mail: magdalena.lemanska@pg.edu.pl

Juan A. Rodríguez-Velázquez

Universitat Rovira i Virgili, Spain

e-mail: juanalberto.rodriguez@urv.cat

Ismael G. Yero

Universidad de Cádiz, Spain

e-mail: ismael.gonzalez@uca.es

A mutual-visibility set of a connected graph G is a set of vertices $S \subset V(G)$ such that for every pair of vertices $x, y \in S$ there is a shortest x, y-path whose interior vertices are not in S. We shall consider a robot navigation model that uses such sets. Assume that in each vertex of a mutual-visibility set S a robot is placed. At each stage one robot moves to a neighbouring vertex. Then, the set S is a mobile mutual-visibility set of G if there exists a sequence of moves of the robots such that all the vertices of G are visited by at least one robot, while keeping all the time the mutual-visibility property for the set of vertices of G occupied by the set of robots. The mobile mutual-visibility number of G is the cardinality of a largest mobile mutual-visibility set of G. These mobile mutual-visibility concepts are introduced in this work, and the study of its combinatorial and computational properties is initiated.

The results of the work are from the article [1]. The speaker is supported by the Spanish Ministry of Science and Innovation, ref. PID2019-105824GB-I00.

References

[1] M. Dettlaff, M. Lemańska, J. A. Rodríguez-Velázquez, I. G. Yero, Mobile mutual-visibility sets in graph. Manuscript, (2024).

ZONAL LABELS GENERALIZED TO ABELIAN GROUPS

John Gimbell

University of Alaska

e-mail: jggimbel@alaska.edu

We consider planar maps with a given abelian group where the vertices are labelled with nonzero elements from the group in such a way that the labels on each region sum to zero. Much interesting work is being done with this concept where the group in question is Z_3 (It is related to the Four Color Theorem). We expand on current ideas and show that some are true, more broadly, in abelian groups in general.

FAIR AND PRIVATE DATA PREPROCESSING THROUGH MICROAGGREGATION

Carlos González

Newcastle University

e-mail: carlos.gonzalez@ncl.ac.uk

Privacy protection for personal data and fairness in automated decisions are fundamental requirements for Responsible Machine Learning [3]. Both may be enforced through data preprocessing and share a common target: data should remain useful for a task, while becoming uninformative of the sensitive information. The intrinsic connection between privacy and fairness implies that modifications performed to guarantee one of these goals, may have an effect on the other, e.g., hiding a sensitive attribute from a classification algorithm might prevent a biased decision rule having such attribute as a criterion. In this talk, we present Fair-MDAV [1], a fairness-and-privacy correcting mechanism based on the MDAV clustering algorithm [2]. This work resides at the intersection of algorithmic fairness and privacy: we show how the two goals are compatible and may be simultaneously achieved, with a small loss in predictive performance. Our results are competitive with both state-of-the-art fairness correcting algorithms and hybrid privacy-fairness methods. Experiments were performed on three widely used benchmark datasets: *Adult Income*, *COMPAS* and *German Credit*.

References

[1] C. González, J. Salas, D. Megías, and P. Missier. Fair and Private Data Preprocessing through Microaggregation. ACM Transactions on Knowledge Discovery from Data 18.3 (2023): 1–24.

[2] Domingo-Ferrer, Josep, and Vicenç Torra. "Ordinal, continuous and heterogeneous k-anonymity through microaggregation." Data Mining and Knowledge Discovery 11 (2005): 195-212.

[3] A. Chouldechova and A. Roth. A snapshot of the frontiers of fairness in machine learning. Communications of the ACM, 63(5):82—89, 2020.

ARC-DISTINGUISHING OF ORIENTATIONS OF GRAPHS

Aleksandra Gorzkowska and Jakub Kwaśny

AGH University of Kraków

e-mail: agorzkow@agh.edu.pl, jkwasny@agh.edu.pl

A distinguishing index of a graph is the minimum number of colours in an edge colouring such that the identity is the only automorphism that preserves the colouring. The study of the distinguishing index was started by Kalinowski and Pilśniak [2] and since then, there have been a number of results on the subject. In particular, the optimal bounds for the distinguishing index have been found for the classes of traceable or claw-free graphs. Recently, the variant of the problem for digraphs has attracted some interest. A distinguishing index of a digraph is the minimum number of colours in an arc colouring that is preserved only by the identity. In particular, results for symmetric digraphs have been obtained [3].

Meslem and Sopena [4] started a study of determining the minimum and maximum value of distinguishing index among all possible orientations of a given graph G. We continue this direction of investigation. However, we take a different approach to the problem and consider the relation between the distinguishing index of the orientations of G and the distinguishing index of G. In the talk, we present sharp results for trees, unbalanced bipartite graphs, traceable graphs and claw-free graphs. With this, we extend the results of Meslem and Sopena to some wider classes of graphs and answer a question posed by them about the class of complete bipartite graphs.

References

[1] A. Gorzkowska, J. Kwaśny, *Arc-distinguishing of orientations of graphs*, arXiv:2402.16169.

[2] R. Kalinowski, M. Pilśniak, *Distinguishing graphs by edge-colourings*, European J. Combin. 45 (2015) 124–131.

[3] R. Kalinowski, M. Pilśniak, M. Prorok, *Distinguishing arc-colourings of symmetric digraphs*, Art Discrete Appl. Math. 6 (2023) #P2.04.

[4] K. Meslem, E. Sopena, *Distinguishing numbers and distinguishing indices of oriented graphs*, Discrete Appl. Math., 285 (2020) 330–342.

ON UNIQUENESS OF PACKING OF TWO AND THREE COPIES OF 2-FACTORS

Igor Grzelec, Monika Pilśniak and Mariusz Woźniak

AGH University of Krakow

e-mail: grzelec@agh.edu.pl, pilsniak@agh.edu.pl, mwozniak@agh.edu.pl

Tomáš Madaras, Alfréd Onderko

P.J. Šafárik University in Košice

e-mail: tomas.madaras@upjs.sk, alfred.onderko@upjs.sk

An *embedding* of a graph G, of order n, (in its complement $\overline{G}$) is a permutation σ on $V(G)$ such that if an edge xy belongs to $E(G)$, then $\sigma(x)\sigma(y)$ does not belong to $E(G)$. In others words, an embedding is an (edge-disjoint) *packing* of two copies of G into a complete graph K_n. At first we will consider the problem of the uniqueness of such packings of two copies. Two such embeddings σ_1, σ_2 of a graph G are said to be *distinct* if the graphs $G \oplus \sigma_1(G)$ and $G \oplus \sigma_2(G)$ are not isomorphic (for graphs G_1 and G_2 with $V(G_1) = V(G_2)$ and $E(G_1) \cap E(G_2) = \emptyset$ the *edge sum* $G_1 \oplus G_2$ has $V(G) = V(G_1) = V(G_2)$ and $E(G) = E(G_1) \cup E(G_2)$). A graph G is called *uniquely embeddable* if for all embeddings σ of G, all graphs $G \oplus \sigma(G)$ are isomorphic.

Let $C_{n_1} \cup C_{n_2} \cup \ldots \cup C_{n_k}$ be a 2-factor *i.e.* a vertex-disjoint union of cycles. We completely characterize 2-factors *i.e.* we prove which 2-factors do not have packing of two copies, which have unique packing of two copies and which have at least two distinct of two copies. During this talk some prove ideas will be presented. Moreover we present the generalization of this problem into the problem of the uniqueness of packing of three copies of 2-factors and give the solution of it.

WHAT COPS N' ROBBERS CAN TELL US ABOUT MARKET HEGEMONIZATION?

Stanisław M. S. Halkiewicz

Wroclaw University of Economics and Business

e-mail: smsh@duck.com

Cops N' Robbers is a popular game, which also plays a vital role in graph theory, where a policeman pursues the criminal. The game is cop-win if cop is able to catch the robber within allowed set of moves, and robber-win, if the robber is able to escape the law indefinitely. Recent advances in graph and game theory provide a toolbox to established whether given game, represented in a graph form, is cop-win or robber-win by scaling down the graph to a solvable form. In this novel approach, the researcher reinterprets the classic game setting, transforming the pursuit-evasion scenario into a strategic competition between a dominant entity, portrayed as the cop, aspiring to establish monopoly in a given market, and a smaller competitor (or rather an aggregation of number of smaller entities), represented as the robber, seeking to persevere, therefore maintaining market diversity. An allowable set of moves is then understood as possible competitive strategies that both players are able to choose. By utilizing the before mentioned tools, one can then determine whether a researched market is likely to be hegemonized by an aspiring monopolist and, if so, approximate the timeframe and counter-strategies. A cop number of such graph can be reinterpreted as minimal number of large players that need to cooperate in order to take over the market.

References

[1] Aigner, M., Fromme, M. (1984). A game of cops and robbers, Discrete Applied Mathematics, Volume 8, Issue 1, p. 1–12.

[2] Bonato, A., Nowakowski, R.J. (1971) The Game of Cops and Robbers on Graphs, Providence, RI: American Mathematical Society.

[3] B.W. Sullivan, N. Townsend ,M. Werzanski (2017). An Introduction to Lazy Cops and Robbers on Graphs, College Mathematics Journal, 48(5), p. 322–333.

ON 3-COLOURABILITY OF ($BULL$, H)-FREE GRAPHS

Nadzieja Hodur, Monika Pilśniak, Magdalena Prorok

AGH University of Kraków

e-mail: nhodur@agh.edu.pl, pilsniak@agh.edu.pl, prorok@agh.edu.pl

and Ingo Schiermeyer

TU Bergakademie Freiberg

e-mail: Ingo.Schiermeyer@math.tu-freiberg.de

We call G an H-free graph, if G does not contain H as an induced subgraph. In a class of *bull*-free graphs, where *bull* is a triangle with two additional edges attached to its two vertices, the 3-colourability problem remains NP-complete. However, in the class of graphs defined by two forbidden subgraphs, *bull* and one of stars $S(1,1,2)$ or $S(1,2,2)$, it is possible to find a polynomial algorithm that resolves 3-colourability. Such an algorithm returns a colouring if the given graph is 3-colourable, or a certain subgraph which is obviously non-3-colourable, otherwise.

In this talk we present such algorithms for (*bull*, $S(1,1,2)$)-free and (*bull*, $S(1,2,2)$)-free graphs. The main tool used is the characterisation of perfect graphs given by the Strong Perfect Graph Theorem.

References

[1] B. Randerath, I. Schiermeyer: Polynomial χ-binding functions and forbidden induced subgraphs: a survey, Graphs Combin. 35 (2019), 1–31.

[2] N. Hodur, M. Pilśniak, M. Prorok, I. Schiermeyer: On 3-colourability of (*bull*, H)-free graphs, arXiv:2404.12515, 2024.

SIMPLE SUBGRAPH TESTING

Andrzej Jastrzębski

Gdańsk University of Technology

e-mail: jendrek@eti.pg.edu.pl

To determine the values of Turan numbers or Ramsey numbers, algorithms are needed to check whether the graph contains a subgraph. A simple automaton will be presented that checks whether there are subgraphs or induced subgraphs in a graph. The results related to Turan numbers and Ramsey numbers will also be presented.

$(2,1)$-GRUNDY COLORING

Nahid Yelene Javier Nol

Universidad Autónoma Metropolitana - Iztapalapa

e-mail: nahid@xanum.uam.mx

J. Fresán-Figueroa, D. González-Moreno and M. Olsen

Universidad Autónoma Metropolitana - Cuajimalpa

e-mail: jfresan@cua.uam.mx, dgonzalez@cua.uam.mx, olsen@correo.cua.uam.mx

An $L(2,1)$-coloring is a vertex coloring where vertices are colored with non-negative integers such that if two vertices are adjacent, then their colors must differ by at least 2, and if two vertices are at distance 2 their colors must be different. The span of an $L(2,1)$-coloring φ is the biggest color used by the coloring φ. The $L(2,1)$-Grundy number is the maximum span among all possible $L(2,1)$-greedy colorings of a graph.

In this talk we present results about the $L(2,1)$-Grundy number for some graph families.

References

[1] T. Calamoneri, The $L(h,k)$-Labelling Problem: An updated Survey and Annotated Bibliography, Comput J. 54-8, (2011) 1344–1371.

[2] G. Chartrand, P. Zhang. Chromatic Graph Theory, in: Discrete Mathematics and its Applications (Boca Raton), CRC Press, Boca Raton, FL, 2009.

[3] J. R. Griggs, R. K. Yeh, Labeling graphs with a condition at distance 2. SIAM J. Discrete Math. 5, (1992) 586—595.

LIST MAJORITY EDGE-COLOURINGS OF GRAPHS

Rafał Kalinowski, Monika Pilśniak and Marcin Stawiski

AGH University of Krakow

e-mail: kalinows@agh.edu.pl, pilsniak@agh.edu.pl, stawiski@agh.edu.pl

A colouring of edges of a graph G is a majority colouring, if for every vertex v of G, at most half the edges incident with v have the same colour. This concept was recently introduced in [1] where, among others, we proved that every finite graph without pendant vertices admits a majority 4-edge colouring. Moreover, if the minimum degree of G is at least 4, then G admits a majority 3-edge colouring.

In the talk, the list version of the problem will be investigated, also for infinite graphs. As a consequence of our results, the Unfriendly Partition Conjecture is confirmed for line graphs.

References

[1] F. Bock, R. Kalinowski, J. Pardey, M. Pilśniak, D. Rautenbach, and M. Woźniak, Majority Edge-Colorings of Graphs, *Electron. J. Combin.* 30(1) 2023 #P1.42.

INTEGRITY OF GRIDS

JULIA KOZIK AND ANDRZEJ ŻAK

AGH University of Kraków

e-mail: julkozik@agh.edu.pl, zakandrz@agh.edu.pl

The integrity of a graph $G = (V, E)$ is defined as the smallest sum $|S| + m(G - S)$, where S is a subset of the set V, and $m(H)$ denotes the order of the largest component of the graph H.

Benko, Ernst, and Lanphier provided and proved an asymptotic bounds for planar graphs in terms of the order of the graph. We prove asymptotic results concerning two-dimensional grid-graphs.

References

[1] D. Benko, C. Ernst, D. Lanphier, Asymptotic bounds on the integrity of graphs and separator theorems for graphs, SIAM Journal on Discrete Mathematics 23 (2009), 265—277.

[2] A. Żak, A note of integrity. Discrete Applied Mathematics 341 (2023), 55—59.

GROUP IRREGULARITY STRENGTH OF DISCONNECTED GRAPHS

Barbara Krupińska and Sylwia Cichacz

AGH University of Kraków

e-mail: bkrupin@agh.edu.pl, cichacz@agh.edu.pl

We investigate the *group irregular strength* ($s_g(G)$) of graphs, i.e the smallest value of s such that for any Abelian group Γ of order s exists a function $g : E(G) \to \Gamma$ such that sums of edge labels at every vertex is distinct. We give results for bound and exact values of ($s_g(G)$) for some chosen families of graphs.

References

[1] M. Anholcer and S. Cichacz, Group irregular labelings of disconnected graphs. Contributions to Discrete Mathematics 12(2) (2017), 158–166.

[2] M. Anholcer and S. Cichacz and M. Milanič, Group irregular strenght of connected graphs. Journal of Combinatorial Optimization 10(3) (2013), 1–17.

[3] T. Nierhoff, A tight bound on the irregularity strength of graphs. SIAM J. Discr. Math 13 (2000), 313-323.

HETEROGENEOUS MOBILE AGENTS IN GRAPHS

Łukasz Kuszner

University of Gdańsk

e-mail: lukasz.kuszner@ug.edu.pl

Computational tasks using teams of mobile agents deployed in a network arise in the context of many applications and theoretically studied problems ranging from two-agent problems like rendezvous to multi-agent scenarios like searching, exploration, patrolling or evacuation.

Agents are often assumed to be identical but scenarios with agents having different capabilities have also been studied in various contexts.

Agents with different speeds were considered in [5], where multiple robots are traveling along a ring to determine their initial positions and in [4, 8] with the goal of patrolling.

In [3] agents capable of traveling in two modes that differ with maximal speeds when searching a line segment were studied.

The problem of evacuating agents with an additional constraint that each type of agent can only use a specific subset of edges in the graph was studied in [1] and the similar approach was applied to the rendezvous problem in [2, 6, 7].

We present an overview of the concept of heterogeneous mobile agents in graphs, the recent results, and open problems.

References

[1] P. Borowiecki, S. Das, D. Dereniowski, and Ł. Kuszner. Distributed evacuation in graphs with multiple exits. In *SIROCCO 2016*, volume 9988 of *LNCS*, pages 228–241, 2016. https://doi.org/10.1007/978-3-319-48314-6_15

[2] S. Cicerone, G. Di Stefano, L. Gąsieniec, and A. Navarra. Asynchronous rendezvous with different maps. In *SIROCCO 2019*, volume 11639 of *LNCS*, pages 154–169. Springer, 2019. https://doi.org/10.1007/978-3-030-24922-9_11

[3] J. Czyżowicz, L. Gąsieniec, K. Georgiou, E. Kranakis, and F. MacQuarrie. The beachcombers' problem: Walking and searching with mobile robots. *Theor. Comput. Sci.*, 608:201–218, 2015. https://doi.org/10.1016/J.TCS.2015.09.011

[4] J. Czyżowicz, L. Gąsieniec, A. Kosowski, and E. Kranakis. Boundary patrolling by mobile agents with distinct maximal speeds. In *ESA 2011*, volume 6942 of *LNCS*, pages 701–712. Springer, 2011. https://doi.org/10.1007/978-3-642-23719-5_59

[5] J. Czyżowicz, E. Kranakis, and E. Pacheco. Localization for a system of colliding robots. In *ICALP 2013*, volume 7966 of *LNCS*, pages 508–519. Springer, 2013. https://doi.org/10.1007/978-3-642-39212-2_45

[6] D. Dereniowski, Ł. Kuszner, and R. Ostrowski. Searching by heterogeneous agents. *J. Comput. Syst. Sci.*, 115:1–21, 2021. https://doi.org/10.1016/j.jcss.2020.06.008

[7] A. Farrugia, L. Gąsieniec, Ł. Kuszner, and E. Pacheco. Deterministic rendezvous with different maps. *J. Comput. Syst. Sci.*, 106:49–59, 2019. https://doi.org/10.1016/j.jcss.2019.06.001

[8] A. Kawamura and Y. Kobayashi. Fence patrolling by mobile agents with distinct speeds. *Distributed Comput.*, 28(2):147–154, 2015. https://doi.org/10.1007/s00446-014-0226-3

ON THE K-METRIC ANTIDIMENSION OF GRAPHS AND ITS APPLICATION TO PRIVACY IN SOCIAL NETWORKS

Elena Fernández, Dorota Kuziak, Manuel Muñoz Márquez and Ismael G. Yero

Universidad de Cádiz, Spain

e-mail: {elena.fernandez,dorota.kuziak,manuel.munoz,ismael.gonzalez}@uca.es

Given a connected graph G, a set $S \subset V(G)$ is a k-*antiresolving set* for G, if k is the largest integer such that for all $u \notin S$ there exists a set $S_u \subseteq V(G) \setminus (S \cup \{u\})$ with $|S_u| \geq k-1$ such that $d_G(u,v) = d_G(x,v)$ for every $v \in S$ and every $x \in S_u$, where $d_G(a,b)$ is the distance between a, b. The k-*metric antidimension* of G is the cardinality of a smallest k-ARS for G.

This work focuses on the use of the k-metric antidimension of graphs as a theoretical framework for the privacy measure of social networks called (k,ℓ)-anonymity. A graph G meets (k,ℓ)-*anonymity* with respect to active attacks to its privacy, if k is the smallest positive integer such that the k-metric antidimension of G is not larger than ℓ.

Graphs with a predetermined structure like cylinders, toruses, and 2-dimensional Hamming graphs, as well as, randomly generated graphs are considered, in order to evaluate the (k,ℓ)-anonymity they achieve. We have taken a combinatorial approach for the graphs with a predetermined structure, whereas for randomly generated graphs we have developed an integer programming formulation and computationally tested its implementation. The results indicated that, according to the (k,ℓ)-anonymity measure, only the 2-dimensional Hamming graphs and some general random dense graphs are achieving some higher privacy properties.

The results of this talk were published in the article [1]. The speaker is supported by the Spanish Ministry of Science and Innovation, ref. PID2019-105824GB-I00.

References

[1] E. Fernández, D. Kuziak, M. Muñoz-Márquez, I. G. Yero, On the (k,l)-anonymity of networks via their k-metric antidimension, Scientific Reports (Nature portfolio) 13 (2023) article # 19090.

MAXIMAL TRANSITIVE SUBTOURNAMENTS OF A DIGRAPH: THE τ OPERATOR

Marisa Gutiérrez and Guadalupe Sánchez-Vallduví

Centro de Matemática de La Plata, FCE-UNLP, Argentina

e-mail: marisa@mate.unlp.edu.ar and guadalupesanchezv@hotmail.com

Bernardo Llano

Universidad Autónoma Metropolitana, Mexico

e-mail: llano@xanum.uam.mx

We introduce the *maximal transitive subtournament* (or the *tt-clique*) operator τ of a digraph D. The τ operator of a digraph D is the intersecting digraph of its tt-cliques preserving the orientation.

This operator is a corresponding notion to the widely studied *clique operator* of graphs (the intersection graph of the maximal complete subgraphs of a given graph). On the other hand, the τ operator is the generalization of the well-known line digraph of a digraph D.

We also define convergent, periodic and divergent digraphs over the τ operator. For the basics on (di)graph operators see [2].

Some basic properties of the operator are studied and we exhibit infinite families of convergent and divergent digraphs under τ. It is proved that for every $p \in \mathbb{N}$ there exists an infinite family of finite τ-periodic digraphs of period p.

References

[1] M. Gutiérrez, B. Llano and G. Sánchez-Vallduví, The maximal transitive subtournaments of a digraph: the τ operator, Matemática Contemporânea 55 (2023), 57–66.

[2] E. Prisner. *Graph dynamics*, Longman, Harlow, 1995.

FIBONACCI CORDIAL LABELING OF CORONA GRAPHS

LUCAS MADER, SARBARI MITRA

Fort Hays State University

e-mail: maderlucasj@gmail.com

An injective function f from vertex set, of a graph G, $V(G)$ to the set $\{F_0, F_1, F_2, \cdots, F_n\}$, where F_i is the i^{th} Fibonacci number $(i = 0, 1, \cdots, n)$, is said to be Fibonacci cordial labeling if the induced function f^* from the edge set $E(G)$ the set $\{0, 1\}$ defined by $f^*(uv) = (f(u) + f(v)) \pmod 2$ satisfies the condition $|e_f(0) - e_f(1)| \leq 1$, where $e_f(0)$ is the number of edges with label 0 and $e_f(1)$ is the number of edges with label 1. A graph that admits Fibonacci cordial labeling is called a Fibonacci cordial graph. In 2020, Mitra and Bhoumik discussed whether the corona graphs $C_n \odot K_m$ for $m \leq 3$ are Fibonacci cordial. We extend their work for $C_n \odot K_m$ for $m \geq 4$ and investigate the conditions under which $K_{n,n,} \odot K_p$ is Fibonacci Cordial.

STUDY OF THE TOTAL TRIPLE ROMAN DOMINATION IN GRAPHS[4]

M. Antonia Mateos-Camacho

University of Seville

e-mail: mantoniamateosc@gmail.com

The Total Triple Roman domination in Graphs arises as a new variant of the Roman domination. A Roman domination in graphs is a modeling of a military defensive problem of the Roman empire defined by Cockayne [3] in 2004. Triple Roman domination was introduced by Ahangar et al. [1] in 2021 with the objective of having each territory defended by three legions, minimizing its cost. Let us consider f as a function $f : V(G) \rightarrow \{0, 1, 2, 3, 4\}$ in the graph $G = (V, E)$, such that, $f(AN[v]) \geq |AN(v)| + 3$ for any $vinV$ with $f(v) < 3$, with $AN(v) \subseteq V$ being the set of adjacent vertices to v with positive label. Total Triple Roman domination was born as a new variant of Triple Roman domination with the aim of making it more efficient in the face of an individual attack on the nodes. This variant defined by a function f on the graph G must satisfy the previous conditions of the Triple Roman domination, in addition to any subgraph induced in G by the set $u \epsilon V$, such that $f(u) \neq 0$ does not have isolated vertices. The Total Triple Roman domination number $\gamma_{[t3R]}$ (G) is defined as the minimum of the weight of the sum of the labels $w(f) = \sum f(v)$ and the function f defined in G is a $\gamma_{[t3R]}$ $(G)-$function. In this work some bounds are established. Exact values are also studied for some families of graphs such as paths, cycles, bistars, bipartite and spider graphs.

References

[1] H. Abdollahzadeh Ahangar, M.P. Álvarez Ruiz, M. Chellali, S.M. Sheikholeslami and J.C. Valenzuela-Tripodoro, *Triple Roman domination in graphs*, Applied Mathematics and Computation. 391 (2021).

[2] R.A. Beeler, T.W. Haynes and S.T. Hedetniemi, *Double Roman domination*. Discrete Appl. Math. 211(2016) 23–29.

[3] E.J. Cockayne, P.A. Dreyer, S.M. Hedetniemi and S.T. Hedetniemi, *Roman domination in graphs*. Discrete Math. 278(2004) 11–22.

[4]*joint work with* Pilar Álvarez-Ruiz, pilar.ruiz@uca.es, J. Carlos Valenzuela-Tripodoro, jcarlos.valenzuela@uca.es

[4] C.S. ReVelle and K.E. Rosing, *Defendens imperium romanum: a classical problem in military strategy.* Amer. Math. Monthly 107 (7) (2000) 585–594.

[5] I. Stewart, *Defend the Roman Empire!* Sci. Amer. 281(6) (1999) 136–139.

NEIGHBOR LOCATING COLORING ON THE PRODUCT OF GRAPHS

Doost Ali Mojdeh

Department of Mathematics, Faculty of Mathematical Sciences, University of Mazandaran

e-mail: damojdeh@umz.ac.ir

Ali Ghanbari

Department of Mathematics, Faculty of Mathematical Sciences, University of Mazandaran

e-mail: ali.ghanbari239@gmail.com

Let G be a graph. A k-coloring of G is a partition $\pi = \{S_1, \cdots, S_k\}$ of $V(G)$ so that each S_i are independent set and take same color. A k-coloring $\pi = \{S_1, \cdots, S_k\}$ of $V(G)$ is a neighbor-locating coloring if any two vertices $u, v \in S_i$, there is a color class S_j for which, one of them has a neighbor in S_j and the other not. The minimum k with this property, is said to be neighbor-locating chromatic number of G, denote by $\chi_{NL}(G)$ of G.
In this talk we discuss on the neighbor-locating chromatic number of Cartesian and lexicographic product of two graphs.

References

[1] L. Alcon, M. Gutierrez, C. Hernando, M. Mora and I.M. Pelayo, Neighbor-locating colorings in graphs, Theoretical Computer Science, 806, (2020) 144-–155.

[2] L. Alcon, M. Gutierrez, C. Hernando, M. Mora, and I. M. Pelayo, The neighbor-locating-chromatic number of pseudotrees, arXiv:1903.11937v1 [math.CO] 28 Mar (2019).

[3] L. Alcon, M. Gutierrez, C. Hernando, M. Mora and I.M. Pelayo, The neighbor-locating-chromatic number of trees and unicyclic graphs. Discussiones Mathematicae Graph Theory, 43 (2023) 659–675, https://doi.org/10.7151/dmgt.2392.

[4] D. A. Mojdeh. On the conjectures of neighbor locating coloring of graphs. Theoretical Computer Science, 922 (2022) 300-–307.

UPPER BOUNDS ON ISOLATION PARAMETERS FOR TREES

Peter Borg

University of Malta

e-mail: peter.borg@um.edu.mt

Magdalena Lemańska

Gdańsk University of Technology

e-mail: magdalena.lemanska@pg.edu.pl

Mercè Mora

Universitat Politècnica de Catalunya

e-mail: merce.mora@upc.edu

María José Souto-Salorio

Universidade da Coruña

e-mail: maria.souto.salorio@udc.es

The concept of isolation in graphs arises by relaxing the condition of domination [1]. Let D be a set of vertices of a graph $G = (V, E)$ and denote by $N[D]$ the set of vertices in D or with a neighbour in D. We say that D is *isolating* if the subgraph induced by $V - N[D]$ has no edges. In general, if $\mathcal{F}$ is a set of graphs, we say that D is $\mathcal{F}$*-isolating* if no subgraph of $G - N[D]$ is a copy of a member of $\mathcal{F}$ [2]. Hence, usual domination and isolation correspond to $\mathcal{F}$-isolation for the sets $\mathcal{F} = \{K_1\}$ and $\mathcal{F} = \{K_2\}$, respectively. In this work, we study $\mathcal{F}$-isolation when $\mathcal{F}$ consists of the k-star $K_{1,k}$ for some $k \geq 1$. Concretely, we establish some upper bounds on the minimum cardinality of a $\{K_{1,k}\}$-isolating set for trees and characterize all trees attaining the given bounds.

References

[1] R. Boutrig, M. Chellali, T.W. Haynes and S. Hedetniemi, Vertex-edge domination in graphs. Aequationes Mathematicae 90 (2016), 355–366.

[2] Y. Caro and A. Hansberg, Partial domination - the isolation number of a graph. Filomat 31(12) (2017), 3925–3944.

STACKED BOOK GRAPHS ARE PERMUTATIONALLY 3-REPRESENTABLE

Khyodeno Mozhui and K. V. Krishna

Indian Institute of Technology Guwahati, India

e-mail: k.mozhui@iitg.ac.in, kvk@iitg.ac.in

A simple graph $G = (V, E)$ is called a word-representable graph if there exists a word w over its vertex set V such that, for all $a, b \in V$, $\overline{ab} \in E$ if and only if a and b alternate in w. The word-representable graphs covers many important classes of graphs including comparability graphs, circle graphs, and 3-colorable graphs. The monograph by Kitaev and Lozin [3] provides a comprehensive account of word-representable graphs, their connections to other contexts, and contributions to the topic.

A word-representable graph is said to be k-word-representable if it is represented by a word in which every letter occurs exactly k times. The smallest k such that a graph is k-word-representable is called the representation number of the graph. If a word representing a word-representable graph is of the form $p_1p_2 \cdots p_k$, where each p_i's is a permutation of its vertices, then the graph is said to be permutationally k-representable. In fact, the class of permutationally representable graphs is precisely the class of comparability graphs [4]. The permutation-representation number (in short *prn*) of a comparability graph is the the minimum value of k such that the graph is permutationally k-representable. It is to be noted that the representation number of a comparability graph is at most its *prn*. The class of complete graphs is precisely the graphs with the *prn* one.

The class of graphs with *prn* at most two is characterized as the class of permutation graphs [1], and the class of circle graphs is characterized as the class of graphs with representation number at most two [2]. In general, it was shown that determining the *prn* and representation number of a permutationally representable graph are computationally hard [5, 2]. In the literature, the *prn* and the representation number for some specific classes of graphs were obtained, in addition to some isolated examples. The classification for the class of graphs with the *prn* at most three is an open problem. In this work, first we reconcile the graphs of with the *prn* at most three. Further, we show that the stacked book graphs have the *prn* as well as the representation number at most three.

References

[1] Tibor Gallai, Transitiv orientierbare Graphen. Acta Math. Acad. Sci. Hungar. 18 (1967), 25–66.

[2] Magnús M. Halldórsson, Sergey Kitaev and Artem Pyatkin, Alternation graphs. Graph-theoretic concepts in computer science. Lecture Notes in Comput. Sci, Springer. 6986 (2011), 191–202.

[3] Sergey Kiteav, and Vadim Lozin, Words and Graphs. Monographs in Theoretical Computer Science. An EATCS Series. Springer, Cham (2015).

[4] Sergey Kitaev and Steve Seif, Word problem of the Perkins semigroup via directed acyclic graphs. Order. 25(3) (2008), 177–194.

[5] Mihalis Yannakakis, The complexity of the partial order dimension problem. SIAM J. Algebraic Discrete Methods. 3(3) (1982), 351–358.

WHICH N-VERTEX E-EDGE GRAPH HAS THE MOST H SUBGRAPHS?

Dániel T. Nagy

HUN-REN Alfréd Rényi Institute of Mathematics

e-mail: nagydani@renyi.hu

We are interested in finding the graph of given size and order that contains the most copies of a certain small graph as a subgraph. Precisely speaking, for a simple graph H, let $ex(n, e, H)$ denote the maximal number of copies of (not necessarily induced) H subgraphs in a graph with n vertices and e edges. There is no theorem telling us the exact value of $ex(n, e, H)$ or even an asymptotic bound for a general H, but the problem is settled for certain specific graphs.

We will overview the theorem of Ahlswede and Katona about $H = K_{1,2}$, its generalization by Reiher and Wagner about any $H = K_{1,s}$ and the speaker's result concerning the case when H is a 4-edge path. For these graphs, $ex(n, e, H)$ is asymptotically achieved by either a clique (if the edge density is high) or the complement of a clique (if the edge density is low). We also discuss a theorem of Alon that describes infinitely many graphs H for which the clique is always the optimal construction.

Blekherman and Patel proved that for any graph H, $ex(n, e, H)$ is asymptotically achieved by a threshold graph. Gerbner, Patkós, Vizer and the speaker showed that for any H this extremal construction is the clique, provided that the edge density is above a threshold c_H. We also investigate a variant of these problems, when the host graph is bipartite.

THE STAR B-CHROMATIC NUMBER OF A GRAPH

Dragana Božović, Iztok Peterin

University of Maribor

e-mail: dragana.bozovic@um.si, iztok.peterin@um.si

Daša Mesarič Štesl

University of Ljubljana

e-mail: dasa.stesl@gmail.com

The b-chromatic number $\chi_b(G)$ of a graph G was introduced by Irving and Manlove [2] in 1999 and is well investigated graph invariant by now. Recently Anholcer et al. [1] in 2022 generalized it to the acyclic b-chromatic number $A_b(G)$ for acyclic colorings of G. We continue with generalization of b-chromatic number to some special colorings, this time in particular to star colorings and we introduce the star b-chromatic number $S_b(G)$ of a graph G.

A star coloring of a graph G is a proper coloring where vertices of every two color classes induce a forest of stars. A strict partial order is defined on the set of all star colorings of G. We introduce, analogue to the b-chromatic number, the star b-chromatic number $S_b(G)$ as the maximum number of colors in a minimum element of the mention order. We present several combinatorial properties of $S_b(G)$, compute the exact value for $S_b(G)$ for several known families and compare $S_b(G)$ with several invariants naturally connected to $S_b(G)$.

References

[1] M. Anholcer, S. Cichacz, and I. Peterin, On b-acyclic chromatic number of a graph. Computational and Applied Mathematics 42(1) (2023), #21.

[2] R. W. Irving and D. F. Manlove, The b-chromatic number of a graph. Discrete Applied Mathematics 91 (1999), 127–141.

AN ALMOST EQUITABLE COLORING OF A WEIGHTED FOREST

Tytus Pikies

Gdańsk University of Technology

e-mail: tytpikie@pg.edu.pl

The talk addressees the problem of equitable coloring of weighted forests. In general, an instance of Equitable Coloring consists of:

- a simple graph (V, E),
- a weight function $w : V \to N$,
- a number of colors m,
- and a question if there exists a coloring of the vertices f, such that for any color c, $\sum_{v \in f^{-1}(c)} w(v) = \sum_{v \in V} w(v)/m$.

One can consider 3 particular cases of the input data.

- When $w \equiv 1$ and the graph is a forest.
- When a graph has no edges and w is an arbitrary function.
- The case when w is an arbitrary function and the graph is a forest.

In the first case the problem is polynomial time. In the second case the problem is NP-complete. However, there exists a polynomial time algorithm (a PTAS, imprecisely speaking) computing an answer that either: there is no such coloring; or that there is coloring f where for any color c, $\sum_{v \in f^{-1}(c)} w(v) \leq (1 + \epsilon) \sum_{v \in V} w(v)/m$, where ϵ is any fixed number greater than 0.

The third case is addressed during the talk. Insights are provided, in particular a classification of the vertices, which can be used to provide a PTAS for Equitable Coloring with respect to weighted forests.

References

[1] B. Baker and E. Coffman Jr., Mutual exclusion scheduling, Theoretical Computer Science, 162(2):225–243, (1996).

[2] D. Hochbaum and D. Shmoys, Using dual approximation algorithms for scheduling problems theoretical and practical results, J. ACM 34(1):144-162, (1987).

RAINBOW TURÁN PROBLEMS[5]

Magdalena Prorok

AGH University of Kraków

e-mail: prorok@agh.edu.pl

One of the central topics in extremal graph theory, known as the Turán problem, is to determine the maximum number of edges of a graph on n vertices that does not contain a copy of a given graph F as a subgraph. Equivalently, the minimum number of edges that forces the existence of F as a subgraph.

In a rainbow version of this problem, for an integer $c \geq 1$ we consider a collection of c graphs $\mathcal{G} = (G_1, \ldots, G_c)$ on a common vertex set, thinking of each graph as edges in a distinct color. We want to force the existence of a rainbow copy of F in $\mathcal{G}$ by having a large number of edges in each graph.

In this talk we present a solution to the problem for directed graphs without rainbow triangles and stars for any number of colors.

References

[1] S. Babiński, A. Grzesik, M. Prorok, Directed graphs without rainbow triangles, arXiv:2308.01461.

[2] D. Gerbner, A. Grzesik, C. Palmer, M. Prorok, Directed graphs without rainbow stars, arXiv:2402.01028.

[5]based on joint work with Sebastian Babiński, Dániel Gerbner, Andrzej Grzesik, Cory Palmer

GRAPHS IN QUANTUM INFORMATION

Monika Rosicka

University of Gdańsk

e-mail: monika.rosicka@ug.edu.pl

Graphs have many applications in quantum information theory. These range from theoretical frameworks for the study of the underlying physical systems and properties to the logistics of building a functional quantum communication network. In this talk we discuss several of these applications.

References

[1] M. Rosicka, R. Ramanathan, P. Gnaciński, K. Horodecki, M. Horodecki, P. Horodecki, S. Severini, *Linear game non-contextuality and Bell inequalities - a graph-theoretic approach*, New Journal of Physics 18 (2016), 045020.

[2] M. Rosicka, P. Mazurek, A. Grudka, M. Horodecki, *Generalized XOR non-locality games with graph description on a square lattice*, Journal of Physics A: Mathematical and Theoretical, vol. 53.(2020).

[3] R. Ramanathan, M. Rosicka, K. Horodecki, S. Pironio, M. Horodecki, P. Horodecki, *Gadget structures in proofs of the Kochen-Specker theorem*, Quantum, (2020), 4, 308-1 - 308-19.

[4] Y. Liu, R. Ramanathan, K. Horodecki, M. Rosicka, P. Horodecki, *Optimal Measurement Structures for Contextuality Applications*, npj Quantum Information volume 9, Article number: 63 (2023)

[5] S. Szarek, M. Rosicka, A. Rutkowski, P. Gnaciński, M. Horodecki, *Constructive nonlocal games with very small classical values*, (arXiv:2112.07741) (2021).

ZERO FORCING IN GRAPHS

Bryan Curtis and Leslie Hogben

Iowa State University

e-mail: bcurtis1@iastate.edu, hogben@aimath.org

<u>Riana Roux</u>

Stellenbosch University

e-mail: rianaroux@gmail.com

Zero forcing is a propagation process on a graph. The propagation process may be describe by the repeated application of the following *colour change rule*: starting with an initial set of blue vertices, a blue vertex v can change the colour of a neighbouring white vertex w to blue if w is the only white neighbour of v. A *zero forcing set* of G is a subset S of vertices such that if S is the initial set of blue vertices the whole graph will eventually be coloured blue. The *zero forcing number* of a graph G, $Z(G)$, is the minimum cardinality of a zero forcing set.

We introduce the idea of *Z-irredundance*, which determines when a zero forcing set is minimal. In this talk we will discuss the relationships between zero forcing and Z-irredundance showcasing similarities and significant differences.

EDGE OPEN PACKING: COMPLEXITY AND COMPUTATIONAL ASPECTS

Boštjan Brešar[1,2] and Babak Samadi[2]

[1]*Faculty of Natural Sciences and Mathematics, University of Maribor, Slovenia*
[2]*Institute of Mathematics, Physics and Mechanics, Ljubljana, Slovenia*

e-mail: bostjan.bresar@um.si, babak.samadi@imfm.si

Given a graph G, two edges $e_1, e_2 \in E(G)$ are said to have a common edge e if e joins an endvertex of e_1 to an endvertex of e_2. A subset $B \subseteq E(G)$ is an edge open packing (EOP) in G if no two edges of B have a common edge in G, and the maximum cardinality of such a set in G is called the edge open packing number, $\rho_e^o(G)$, of G. In this paper, we prove that the decision version of the EOP number is NP-complete even when restricted to graphs with universal vertices and Eulerian bipartite graphs, respectively. In contrast, we present a linear-time algorithm that computes the parameter for trees. We also solve a problem posed in an earlier paper on this topic. Notably, we characterize the graphs G that attain the upper bound $\rho_e^o(G) \leq |E(G)|/\delta(G)$.

This problem was introduced in [3]. Note that the EOP sets are color classes of the injective edge coloring of graphs ([2]) as well as a generalization of induced matchings ([1, 4]).

1 Main results

We first discuss the following decision problem associated with the EOP number.

Edge open packing problem	
Instance: A graph G and an integer $k \leq \lvert E(G)\rvert$.	(1)
Question: Is $\rho_e^o(G) \geq k$?	

We show that the problem (1) is NP-complete for some special families of graphs.

Theorem 1 Edge Open Packing Problem *is NP-complete even for graphs with universal vertices.*

Moreover, we prove that (1) is in some sense harder than Independent Set Problem as it is known that the independence number of bipartite graphs can be computed in polynomial time.

Theorem 2 EDGE OPEN PACKING PROBLEM *is NP-complete even for Eulerian bipartite graphs.*

We prove that the EOP number and an optimal EOP set for any tree T can be computed/constructed in linear time by exhibiting an efficient algorithm for EOP in trees, in which the parameter can be computed in terms of four auxiliary versions of the EOP number which are recursively defined based on rooted trees.

Theorem 3 *There exists a linear-time algorithm for computing the edge open packing number of a tree.*

We define the family $\mathcal{F}$ as follows. Let G be a bipartite graph of minimum degree $k \geq 2$ with partite sets $A \cup C$ and B such that every vertex in B has 1 and $k-1$ neighbors in A and C, respectively.

Theorem 4 *For any graph G of size m, $\rho_e^o(G) = m/\delta(G)$ if and only if either G is a disjoint union of stars or $G \in \mathcal{F}$.*

References

[1] K. Cameron, Induced matchings. Discrete Applied Mathematics 24 (1989), 97–102.

[2] D.M. Cardoso, J.O. Cerdeira, J.P. Cruz and C. Dominic, Injective edge coloring of graphs. Filomat 33 (2019), 6411–6423.

[3] G. Chelladurai, K. Kalimuthu and S. Soundararajan, Edge open packing sets in graphs. RAIRO - Operations Research 56 (2022), 3765–3776.

[4] L.J. Stockmeyer and V.V. Vazirani, NP-completeness of some generalizations of the maximum matching problem. Information Processing Letters 15 (1982), 14–19.

PATH ISOLATION IN GRAPHS

Karl Bartolo, Peter Borg and Dayle Scicluna

University of Malta

e-mail: karl.bartolo.16@um.edu.mt, peter.borg@um.edu.mt,
dayle.scicluna.09@um.edu.mt

Given a graph G and a set $\mathcal{F}$ of graphs, the $\mathcal{F}$-isolation number is the size of a smallest subset D of the vertex set of G such that $G - N[D]$ (the graph obtained from G by removing the closed neighbourhood of D) does not contain a copy of a graph in $\mathcal{F}$. The path isolation number $\iota(G, P_i)$ for $i > 0$ has attracted particular interest among graph theorists. For $i = 1$, since $P_1 = K_1$, we have Ore's (1962) result [4] that $\gamma(G) = \iota(G, P_1) \leq \frac{n}{2}$ where $\gamma(G)$ is the domination number of an n-vertex connected graph G. For $i = 2$, since $P_2 = K_2$, we have Caro and Hansberg's (2017) result [2] that $\iota(G, P_2) \leq \frac{n}{3}$ provided G is connected but not a 5-cycle or a 2-clique. For $i = 3$, it was shown by Zhang and Wu [5], and independently and in a stronger form by Borg [1], that $\iota(G, P_3) \leq \frac{2n}{7}$ unless $G \in \{P_3, C_3, C_6\}$. This can be improved to $\frac{n}{4}$ if G is not a $\{P_3, C_7, C_{11}\}$-graph and the girth is at least 7. Recently Huang, Zhang and Jin [3] showed that for a connected graph G that has no 6-cycles or has no induced 5- and 6-cycles, then $\iota(G, P_3) \leq \frac{n}{4}$ provided G is not a $\{P_3, C_3, C_7, C_{11}\}$-graph. Joint work with Bartolo and Borg improved this result for subcubic graphs. If G is subcubic and has no induced 6-cycles, then $\iota(G, P_3) \leq \frac{n}{4}$ provided G is not one of twelve specific graphs. The bound is sharp.

References

[1] P. Borg, Isolation of connected graphs, Discrete Appl. Math. 339 (2023), 154–165.

[2] Y. Caro and A. Hansberg, Partial domination - the isolation number of a graph, Filomat 31:12 (2017), 3925–3944.

[3] Y. Huang, G. Zhang and X. Jin, New results on the 1-isolation number of graphs without short cycles, arXiv:2308.00581.

[4] O. Ore, Theory of graphs, American Mathematical Society Colloquium Publications, vol. 38, American Mathematical Society, Providence, R.I., 1962.

[5] G. Zhang and B. Wu, $K_{1,2}$-isolation in graphs, Discrete Applied Mathematics 304 (2021), 365–374.

STRONG PROPER VERTEX CONNECTION IN DIGRAPHS

Elżbieta Sidorowicz

University of Zielona Góra

e-mail: e.sidorowicz@wmie.uz.zgora.pl

Yingbin Ma, Kairui Nie, Mingli Wang

Henan Normal University

e-mail: mayingbincw@htu.cn, niekairui@163.com, wangmingli@stu.htu.edu.cn

An vertex-colored path is vertex proper if it does not contain two adjacent vertices with the same color. An vertex-colored digraph D is properly vertex connected if, between every ordered pair of vertices, there is a directed proper path. A vertex-colored digraph D is strong properly vertex connected if there exists a vertex proper geodesic between any ordered pair of vertices. The smallest number of colors needed to make D (strong) properly vertex connected is called the (strong) proper vertex connection number of D. The proper vertex connection number and strong proper vertex connection number of D is denoted by $\overrightarrow{pvc}(D)$ and $\overrightarrow{spvc}(D)$, respectively.

It is known that the proper vertex connection number of any strong digraph is at most 3 ([1]). However, the strong proper vertex connection number can be arbitrarily large ([2]). In this talk, we will provide some properties of the strong properly connected vertex-coloring. Additionally, we will present upper bounds on the strong proper vertex connection number for some classes of digraphs.

References

[1] G. Ducoffe, R. Marinescu-Ghemeci, A. Popa, On the (di)graphs with (directed) proper connection number two, Discrete Appl. Math. 281 (2020), 203-215.

[2] K. Nie, Y. Ma, E. Sidorowicz, (Strong) Proper vertex connection of some digraphs. Appl. Math. Comput. 458 (2023), 128243.

VARIETY OF GENERAL POSITION PROBLEMS IN GRAPHS

Jing Tian

Zhejiang University of Science and Technology, University of Ljubljana

e-mail: ingtian526@126.com

Let X be a vertex subset of a graph G. Two vertices $u, v \in V(G)$ are X-positionable if $V(P) \cap X \subseteq \{u, v\}$ holds for any shortest u,v-path P. If every pair of vertices from X are X-positionable, then X is called a general position set. The general position number of G is the cardinality of a largest general position set of G, and this concept has been already well investigated. In this talk, I will introduce varieties of general position problems based on which natural pairs of vertices are required to be X-positionable. This yields the definition of the total (resp. dual, outer) general position number. I will demonstrate that the total general position sets coincide with sets of simplicial vertices, and that the outer general position sets coincide with sets of mutually maximally distant vertices. Additionally, I will show that a general position set is a dual general position set if and only if its complement is convex. Furthermore, I will present results on the total general position number, the outer general position number, and the dual general position number for arbitrary Cartesian products of graphs.

DOMINATION NUMBER OF THE CARTESIAN PRODUCT OF A PATH WITH ANY PAIR OF GRAPHS

Omar Tout

Sultan Qaboos University

e-mail: o.tout@squ.edu.om

For a simple finite graph G, let $V(G)$ denote the set of vertices of G. We say that a vertex $u \in V(G)$ dominates a vertex v if $u = v$ or v is adjacent to u. A dominating set of G, is a subset of vertices of G which dominates all the vertices of G. The domination number of G, denoted $\gamma(G)$, is the size of a smallest dominating set of G. The Cartesian product $X \square Y$ of two graphs X and Y is the graph whose vertex set is $V(X) \times V(Y)$ and edge set defined as follows. Two vertices (x_1, y_1) and (x_2, y_2) are adjacent in $X \square Y$ if either $x_1 = x_2$ and y_1 and y_2 are adjacent in Y, or $y_1 = y_2$ and x_1 and x_2 are adjacent in X. The still open conjecture of Vizing, see [1], states that $\gamma(X \square Y) \geq \gamma(X)\gamma(Y)$ for any pair of graphs X and Y.

In 2000, Clark and Suen showed in [2] that $\gamma(X \square Y) \geq \frac{1}{2}\gamma(X)\gamma(Y)$ for any pair of graphs X and Y. To this date, $\frac{1}{2}$ remains the best obtained coefficient towards proving Vizing's conjecture. Clark and Suen's result implies that $\gamma(X \square Y \square Z) \geq \frac{1}{4}\gamma(X)\gamma(Y)\gamma(Z)$ for any triple of graphs X, Y and Z. We show that this lower bound can be improved for special graphs. In particular, for any $n \geq 1$, we show that $\gamma(X \square Y \square P_n) \geq c_n \gamma(X)\gamma(Y)\gamma(P_n)$ where c_n is almost $\frac{3}{4}$ when n is big enough. Our proof can be found in [3]. It uses space projections and follows the new framework to approach Vizing's conjecture which appeared in [4].

References

[1] V. G. Vizing, *Some unsolved problems in graph theory*, Russian Math. Surveys 23(6) (1968) 125-141.

[2] W. E. Clark and S. Suen, *An inequality related to Vizing's conjecture*, Electronic Journal of Combinatorics 7 (2000) # N4.

[3] O. Tout, *On the domination number of the cartesian product of the path graph P_2 and a pair of graphs*, arXiv, 2312.09208 (2023). `https://arxiv.org/abs/2312.09208`

[4] B. Brešar, B. L. Hartnell, M. A. Henning, K. Kuenzel and D. F. Rall, *A new framework to approach Vizing's conjecture*, Discussiones Mathematicae Graph Theory. 41(3) (2021) 749-762.

THE ALMOST MAJORITY NEIGHBOR SUM DISTINGUISHING EDGE COLORING

Elżbieta Sidorowicz and Elżbieta Turowska

University of Zielona Góra

e-mail: e.sidorowicz@im.uz.zgora.pl, e.turowska@im.uz.zgora.pl

A k-edge coloring of a graph with colors in $[k]$ is neighbor sum distinguishing if, for any two adjacent vertices, the sums of the colors of the edges incident with each of them are distinct. The smallest value of k such that a neighbor sum distinguishing k-coloring of G exists is denoted by $\chi^{e}_{\sum}(G)$. When we add the additional restriction that the edge k-coloring must be proper, then the smallest value of k such that such a coloring exists is denoted by $\chi'_{\sum}(G)$. The first type of coloring is related with the 1-2-3 Conjecture, which was already proven by Keusch ([2]). The second type of coloring is related with another conjecture proposed by Flandrin et al. ([1]), which states that $\chi'_{\sum}(G) \leq \Delta(G) + 2$ for any graph G with no components isomorphic to K_2 and $G \neq C_5$. This conjecture remains open.

We consider an edge coloring that is on one hand stronger than the edge coloring in the 1-2-3 Conjecture, and on the other hand weaker than the coloring in conjecture proposed by Flandrin et al.. An edge k-coloring of a graph G is called almost majority if for every vertex $v \in V(G)$ and every color $\alpha \in [k]$ at most $\lceil d(v)/2 \rceil$ edges incident to v have the color α. An edge k-coloring of a graph G is called almost majority neighbor sum distinguishing if it is almost majority and neighbor sum distinguishing. The minimum value of k for which there exists such an edge k-coloring of a graph G is called the almost majority neighbor sum distinguishing index of a graph G and is denoted by $\chi^{AM}_{\sum}(G)$. We study $\chi^{AM}_{\sum}(G)$ for some classes of graphs.

References

[1] E. Flandrin, A. Marczyk, J. Przybyło, J-F. Sacle, M. Woźniak, Neighbour sum distinguishing index. Graphs Combin. 29(5) (2013), 1329–1336.

[2] R. Keusch, A Solution to the 1-2-3 Conjecture. J. Combin. Theory Ser. B 166 (2024) 182–202.

BACKBONE COLORING: CURRENT PROGRESS AND OPEN PROBLEMS

Krzysztof Turowski

Jagiellonian University

e-mail: krzysztof.szymon.turowski@gmail.com

The concept of backbone coloring, a variant of the classic graph coloring problem, has garnered significant attention due to its theoretical appeal and practical applications in areas such as frequency assignment, scheduling, and network design. This talk will provide an overview of recent progress in backbone coloring, highlighting key advancements and methodologies. Additionally, the talk will address the most promising and stubborn remaining open problems and challenges. In particular, we will focus on the coloring of complete graphs with tree backbones and bounded-degree graphs with tree, path, and matching backbones.

ROMAN DOMINATION ON FUZZY GRAPHS

Juan Carlos Valenzuela-Tripodoro

University of Cádiz (Spain)

e-mail: jcarlos.valenzuela@uca.es

Martin Cera, Pedro Garcia-Vázquez

University of Sevilla (Spain)

e-mail: mcera@us.es, pgvazquez@us.es

We make a contribution to the well-known problem of Roman domination in graph theory as it relates to fuzzy graphs. Domination in fuzzy graphs has been studied using a variety of approaches. By taking into account just effective edges, Somasundaram and Somasundaram [1] examined domination and total domination in fuzzy graphs. Domination in fuzzy graphs was introduced by Nagoor Gani and Chandrasekaran [2] as the number of vertices in a dominating set that makes use of strong edges. Based on the weight of strong edges, Manjusha and Sunitha [4] determined the domination number of fuzzy graphs. Moreover, they used this idea to examine a fuzzy graph's strong node covering number [5].

We will use the weights of strong edges to define the Roman domination number for a fuzzy graph, based on the domination concept put forth by Manjusha and Sunitha. Cockayne et al. [3], who took their cue from a historical defensive tactic ascribed to the reign of Emperor Constantine I The Great (see [6]), are credited for establishing Roman dominance in graphs. This tactic required that every weak point in the Roman Empire have a neighboring fortress (having two legions) that could send a legion to defend it in case of an unexpected attack. This guaranteed that the more powerful city would not have to jeopardize its own security in order to send reinforcements to defend the beleaguered area.

This paper presents the notions of strong-neighbors Roman domination function/number of a fuzzy graph and shows how it relates to other well-known domination parameters. For particular fuzzy graphs, we derive bounds, find the strong-neighbors Roman domination number, and describe the fuzzy graphs for which extreme values are obtained.

References

[1] A. Somasundaram, S. Somasundaram. Domination in fuzzy graphs I, Pattern Recognition Lett. 19 (1998) 787—791.

[2] A. Nagoor Gani, V.T. Chandrasekaran. Domination in fuzzy graph, Adv. Fuzzy Sets Syst. 1 (1) (2006) 17—26.

[3] E.J. Cockayne, P.A. Dreyer, S.M. Hedetniemi, S.T. Hedetniemi. Roman domination in graphs, Discrete Math. 278 (2004) 11—22.

[4] O.T. Manjusha, M.S. Sunitha. Strong Domination in Fuzzy Graphs, Fuzzy Inf. Eng. 7 (2015) 369—377.

[5] O.T. Manjusha, M.S. Sunitha. Coverings, matchings and paired domination in fuzzy graphs using strong arcs, Iran. J. Fuzzy Syst. 16 (1) (2019) 145—157.

[6] I. Stewart. Defend the Roman Empire!, Sci. Amer. 281 (6) (1999) 136—139.

ABOUT UNIVERSAL γ_2-FIXERS TREES

Mercé Mora

Universitat Politécnica de Catalunya, Spain

e-mail: merce.mora@upc.edu

María Luz Puertas

Universidad de Almeria, Spain

e-mail: mpuertas@ual.es

Rita Zuazua

Universidad Nacional Autónoma de México, Mexico

e-mail: ritazuazua@ciencias.unam.mx

A set of vertices D of a graph G is a *distance 2-dominating* set of G if the distance between each vertex $u \in (V(G) - D)$ and D is at most two. Let $\gamma_2(G)$ denote the size of a smallest distance 2-dominating set of G.

For any permutation π of the vertex set of G, the *prism of* G *with respect to* π is the graph πG obtained from two copies G_1 and G_2 of G by joining $u \in V(G_1)$ and $v \in V(G_2)$ if and only if $v = \pi(u)$. If $\gamma_2(\pi G) = \gamma_2(G)$ for any permutation π of $V(G)$, then G is called a universal γ_2-fixer. In this work we study the property to be universal γ_2-fixers for trees.