ON OPERATIONS PRESERVING WORD-REPRESENTABILITY OF GRAPHS

Tithi Dwary and K. V. Krishna

Indian Institute of Technology Guwahati, India

e-mail: tithi.dwary@iitg.ac.in, kvk@iitg.ac.in

A simple graph is called a word-representable graph if there is a word over its vertex set such that any two vertices are adjacent in the graph if and only if they alternate in the word. The class of word-representable graphs was first studied by Sergey Kitaev and Steven Seif in the context of Perkin semigroup [4]. Over the years an extensive literature has developed on this topic, impacting various fields of mathematics and computer science. A word-representable graph is a k-word-representable graph, if it is represented by a word in which every letter appears exactly k times. The smallest k such that a graph is k-word-representable is said to be the representation number of the graph. A word-representable graph is said to be a permutationally representable graph, if it can be represented by a word that is a concatenation of permutations on their vertices. The class of comparability graphs, graphs which admit transitive orientations, is precisely the class of permutationally representable graphs [4]. The smallest k such that a permutationally representable graph is represented by a concatenation of k permutations on its vertices is called the permutation-representation number (in short, *prn*) of the graph. Further, it is known that the general problems of determining the *prn* of a permutationally representable graph, and the representation number of a word-representable graph are computationally hard. For a detailed introduction to this topic, one may refer to the monograph [3].

The graph operations were proved to be useful for determining the representation number of graphs. For example, it was proved in [5] that 3-subdivision of every graph is 3-word-representable and utilizing this, the representation number of prism is determined. It was proved in [3] that the class of word-representable graphs is closed under certain graph operations such as connecting two graphs by an edge, and gluing two graphs at a vertex. Moreover, the representation numbers of the resulting graphs were obtained. Some fundamental graph operations viz., edge-deletion, edge addition do not necessarily preserve the word-representability; however, certain sufficient conditions on graphs to preserve word-representability with respect to these operations were established [1].

In this work, we obtain necessary and sufficient conditions for permutation representability of graphs with respect to the following operations: gluing two graphs at a vertex, replacing a vertex by a module, and lexicographical product of graphs. Further, we obtain the *prn*s of the resultant graphs. A modular decomposition of a graph is a partition of the vertex set of the graph into modules. While it was introduced to study the structure of comparability graphs, it has applications in the theory of posets, and scheduling problems. In this work, we extend the characterization of comparability graphs with respect to the modular decomposition (given in [6]) to word-representable graphs. Accordingly, we determine the representation number of a word-representable graph in terms of the *prn*s of its modules and the representation number of the quotient graph. In this connection, we also obtain a complete answer to the open problem posed by Kitaev and Lozin [3, Chapter 7] on the word-representability of the lexicographical product of graphs.

References

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