INTERVAL COLOURING THICKNESS VIA THE ERDŐS FAMILY OF GRAPHS

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Let $\theta_{int}(G)$ denote the minimum number of parts in a partition of the edge set of the graph G such that graphs induced by all the parts are interval colourable. Giving a finite projective plane $\pi(n)$ of order n with the sets W,L of points and lines, respectively, $Erd(n)$ is known to be a graph with vertex set $W \cup L \cup \{u\}$ and edge set $\{wl: \ w \in W, l \in L$ and w is incident to l $\} \cup \{ul : l \in L\}$. Next, if $L = \{l_1, \ldots, l_{n^2+n+1}\}$ and a sequence $r_1, \ldots, r_{n^2+n+1}$ of positive integers is given, by $Erd(r_1, \ldots, r_{n^2+n+1})$ we mean a graph resulting from $Erd(n)$ by a multiplication of the vertex l_i with r_i vertices made for all $i \in [n^2+n+1]$. Graphs $Erd(r_1, \ldots, r_{n^2+n+1})$ constructed for all possible finite projective planes and all possible parameters $r_1, \ldots, r_{n^2+n+1}$ form the Erdős family of graphs.

Let l be a fixed line of a finite projective plane $\pi(n)$ and $w_1, \ldots, w_{n+1}$ be all points incident to l. Next let for $i \in [n+1]$, a set L_i consist of all lines incident to w_i that are different from l. We prove that if an ordering $l_1, \ldots, l_{n^2+n+1}$ of the set L is given and positive integers $r_1, \ldots, r_{n^2+n+1}$ are such that at least t different indices i from $[n+1]$ satisfy $r_k = r_j$ if $l_k, l_j \in L_i$, then

$$\theta_{int}(Erd(r_1, \ldots, r_{n^2+n+1})) \leq \max\Big\{2, \Big\lceil \frac{n+2-t}{2} \Big\rceil \Big\}.$$

Consequently, a tight upper bound of $\left\lceil \frac{n+2}{2} \right\rceil$ on $\theta_{int}(Erd(r_1, \ldots, r_{n^2+n+1}))$ is valid in the whole Erdős family of graphs.

References

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